Example 7 Find the term independent of x in the expansion of $\left(\dfrac{\sqrt{x}}{\sqrt{3}} + \dfrac{\sqrt{3}}{2x^2}\right)^{10}$.

Solution Let $(r+1)^{\text{th}}$ term be independent of x which is given by

$$T_{r+1} = {}^{10}C_r \left(\sqrt{\frac{x}{3}}\right)^{10-r} \left(\frac{\sqrt{3}}{2x^2}\right)^r$$

$$= {}^{10}C_r \frac{x^{\frac{10-r}{2}}}{3^{\frac{10-r}{2}}} \, 3^{\frac{r}{2}} \, \frac{1}{2^r x^{2r}}$$

$$= {}^{10}C_r \, 3^{\frac{r}{2} - \frac{10-r}{2}} \, 2^{-r} \, x^{\frac{10-r}{2} - 2r}$$

Since the term is independent of x, we have

$$\frac{10-r}{2} - 2r = 0 \qquad \Rightarrow \qquad r = 2$$

Hence 3rd term is independent of x and its value is given by

$$T_3 = {}^{10}C_2 \frac{3^{-3}}{4} = \frac{10 \times 9}{2 \times 1} \times \frac{1}{9 \times 12} = \frac{5}{12}$$