Example 5 Find the coefficient of x^{11} in the expansion of $\left(x^3 - \frac{2}{x^2}\right)^{12}$

Solution Let the general term, i.e., $(r + 1)^{th}$ contain x^{11}.

We have
$$T_{r+1} = {}^{12}C_r\,(x^3)^{12-r}\left(-\frac{2}{x^2}\right)^r$$
$$= {}^{12}C_r\,x^{36-3r-2r}\,(-1)^r\,2^r$$
$$= {}^{12}C_r\,(-1)^r\,2^r\,x^{36-5r}$$

Now for this to contain x^{11}, we observe that

$$36 - 5r = 11, \text{ i.e., } r = 5$$

Thus, the coefficient of x^{11} is

$${}^{12}C_5\,(-1)^5\,2^5 = -\frac{12\times11\times10\times9\times8}{5\times4\times3\times2}\times32 = -25344$$