Example 4 Evaluate: $\left(x^2 - \sqrt{1-x^2}\right)^4 + \left(x^2 + \sqrt{1-x^2}\right)^4$

Solution Putting $\sqrt{1-x^2} = y$, we get

The given expression $= (x^2 - y)^4 + (x^2 + y)^4 = 2\ [x^8 + {}^4C_2\ x^4\ y^2 + {}^4C_4\ y^4]$

$$= 2\left[x^8 + \frac{4 \times 3}{2 \times 1} x^4 \cdot (1-x^2) + (1-x^2)^2 \right]$$

$$= 2\ [x^8 + 6x^4\ (1-x^2) + (1-2x^2 + x^4]$$

$$= 2x^8 - 12x^6 + 14x^4 - 4x^2 + 2$$