Example 13 Which of the following is larger?

$99^{50} + 100^{50}$ or 101^{50}

We have $(101)^{50} = (100 + 1)^{50}$

$$= 100^{50} + 50\,(100)^{49} + \frac{50.49}{2.1}\,(100)^{48} + \frac{50.49.48}{3.2.1}(100)^{47} + \ldots (1)$$

Similarly $99^{50} = (100 - 1)^{50}$

$$= 100^{50} - 50\,.\,100^{49} + \frac{50.49}{2.1}\,(100)^{48} - \frac{50.49.48}{3.2.1}(100)^{47} + \ldots (2)$$

Subtracting (2) from (1), we get

$$101^{50} - 99^{50} = 2\left[50 \cdot (100)^{49} + \frac{50 \cdot 49 \cdot 48}{3 \cdot 2 \cdot 1}100^{47} + \ldots\right]$$

$$\Rightarrow \quad 101^{50} - 99^{50} = 100^{50} + 2\left[\frac{50 \cdot 49 \cdot 48}{3 \cdot 2 \cdot 1}\,100^{47} + \ldots\right]$$

$$\Rightarrow \quad 101^{50} - 99^{50} > 100^{50}$$

Hence $101^{50} > 99^{50} + 100^{50}$