Example 2 Expand the following $(1 - x + x^2)^4$

Solution Put $1 - x = y$. Then

$$
\begin{aligned}
(1 - x + x^2)^4 &= (y + x^2)^4 \\
&= {}^4C_0 \; y^4 (x^2)^0 + {}^4C_1 \; y^3 (x^2)^1 \\
&\quad + {}^4C_2 \; y^2 (x^2)^2 + {}^4C_3 \; y (x^2)^3 + {}^4C_4 (x^2)^4 \\
&= y^4 + 4y^3 x^2 + 6y^2 x^4 + 4y x^6 + x^8 \\
&= (1 - x)^4 + 4x^2 (1 - x)^3 + 6x^4 (1 - x)^2 + 4x^6 (1 - x) + x^8 \\
&= 1 - 4x + 10x^2 - 16x^3 + 19x^4 - 16x^5 + 10x^6 - 4x^7 + x^8
\end{aligned}
$$