Question3

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Question 3. In ΔABC, area of $\Delta ABC = 18$, area of $\Delta BDF = 2$, length of $DF = 2\sqrt{2}$. Find the circumradius of ΔABC.

Here, D & F are the feet of altitudes from A & C respectively

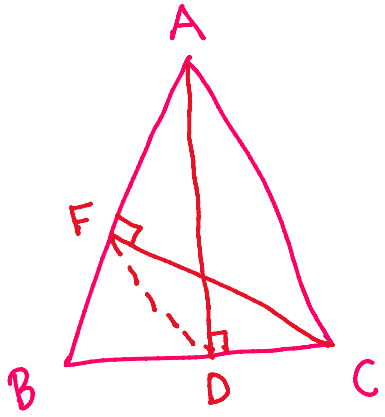
(a) $\frac{9}{2}$

(b) $\frac{8}{5}$

(c) $\frac{7}{4}$

(d) 1

Solution:→



We know that the area of a triangle is:→

$$\text{Area} = \frac{1}{2}ab\sin C$$

$\therefore$ area of $\Delta BDF = \frac{1}{2} \times BD \times DF \times \sin B$

$$2 = \frac{1}{2} \times c\cos B \times a\cos B \times \sin B$$

$$\Rightarrow \quad ac\sin B \cdot \cos^2 B = 4 \quad \text{—— (1)}$$

Now, area of $\Delta ABC = 18$

$$\Rightarrow \quad \frac{1}{2}ac\sin B = 18$$

$$\Rightarrow \quad ac\sin B = 36 \quad \text{—— (2)}$$

Now, dividing equation (1) by equation (2):→

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$$\cos^2 B = \frac{4}{36} = \frac{1}{9}$$

$$\Rightarrow \quad \cos B = \frac{1}{3} \quad \text{—— (3)}$$

Now, using the formulae of side of pedal triangle:→

$$\Rightarrow \quad DF = b\cos B$$

$$2\sqrt{2} = b \times \frac{1}{3}$$

$$\Rightarrow \quad b = 6\sqrt{2} \quad \text{—— (4)}$$

Now, from equation (2), $ac = \dfrac{36}{2\sin B}$

$$= \frac{36}{\sqrt{1-\cos^2 B}}$$

$$= \frac{36}{\sqrt{1-\frac{1}{9}}}$$

$$= \frac{36}{2\sqrt{2}/3}$$

$$\Rightarrow \quad ac = \frac{54}{\sqrt{2}} \quad \text{—— (5)}$$

Now, we know that,

$$\text{Circumradius} = R = \frac{abc}{4\Delta}$$

$\Rightarrow$ $$R = \frac{a c \times b}{4 \times \Delta}$$

$$= \frac{\frac{54}{\sqrt{2}} \times 6\sqrt{2}}{4 \times 18}$$

$\Rightarrow$ $$\boxed{R = \frac{9}{2}}$$ Ans.

(Option (a))