Question 1
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Question 1. The perimeter of a pedal triangle is:→

(a) $\frac{2Rs}{r}$

(b) $\frac{2s^2}{rR}$

(c) $\frac{2rs}{R}$

(d) $\frac{2rR}{s^2}$

Solution:-

Perimeter = Sum of the three sides of the triangle

We know that the sides of a pedal triangle are $a\cos A$, $b\cos B$ & $c\cos C$.

$\therefore$ Perimeter $= a\cos A + b\cos B + c\cos C$ ——— (1)

Now, we know from Sine law that:→

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$$

$\therefore$ We can write equation (1) as:→

$$\text{Perimeter} = 2R\sin A\cdot\cos A + 2R\sin B\cdot\cos B + 2R\sin C\cdot\cos C$$

$$= R[2\sin A\cos A + 2\sin B\cos B + 2\sin C\cos C]$$

$$= R[\sin 2A + \sin 2B + \sin 2C] \quad \text{——— (2)}$$

using the formula of $\sin 2A = 2\sin A\cos A$

Now, using

$$\sin\alpha + \sin\beta = 2\sin\left(\frac{\alpha+\beta}{2}\right)\cdot\cos\left(\frac{\alpha-\beta}{2}\right)$$

we can write equation (2) as:→

$$\text{Perimeter} = R\left[2\sin\left\{\frac{2A+2B}{2}\right\}\cos\left\{\frac{2A-2B}{2}\right\} + \sin 2C\right]$$

$$= R\left[2\sin(A+B)\cos(A-B) + \sin 2C\right]$$

$$= R\left[2\sin(\pi-C)\cos(A-B) + \sin 2C\right]$$ {Since $A+B+C=\pi \Rightarrow A+B=\pi-C$}

$$= R\left[2\sin C\cos(A-B) + 2\sin C\cos C\right]$$

$$= 2R\sin C\left[\cos(A-B) + \cos\{\pi-(A+B)\}\right]$$ {$\because A+B+C=\pi \Rightarrow C=\pi-(A+B)$}

$$= 2R\sin C\left[\cos(A-B) - \cos(A+B)\right] \quad \text{—— (3)}$$

Now using

$$2\sin\alpha\sin\beta = \cos(\alpha-\beta) - \cos(\alpha+\beta),$$

we can write equation (3) as:→

$$\text{Perimeter} = 2R\sin C \times 2\sin A\sin B$$

$$= 4R\sin A\cdot\sin B\cdot\sin C$$

$$= 4R \times \frac{a}{2R} \times \frac{b}{2R} \times \frac{c}{2R}$$ {Using Sine Law}

$$= \frac{abc}{2R^2}$$

$$= \frac{2}{R} \times \frac{abc}{4R}$$

Now, $\boxed{\text{Area of Triangle} = \Delta = \frac{abc}{4R}}$

$\therefore$ Perimeter $= \frac{2}{R} \times \Delta$ ——— (4)

Now, we know that

$$\boxed{\text{Inradius} = r = \frac{\Delta}{s}}$$

$$\Rightarrow \Delta = r \cdot s$$

$\therefore$ From equation (4) we can write as: →

$$\text{Perimeter} = \frac{2}{R} \times r \cdot s$$

$$\Rightarrow \text{Perimeter of pedal triangle} = \frac{2r \cdot s}{R}$$ Ans.

(Option C)