

3.10 COMBINATION OF RESISTORS – SERIES AND PARALLEL

The current through a single resistor R across which there is a potential difference V is given by Ohm's law $I = V/R$. Resistors are sometimes joined together and there are simple rules for calculation of equivalent resistance of such combination.



FIGURE 3.13 A series combination of two resistors R_1 and R_2 .

Two resistors are said to be in *series* if only one of their end points is joined (Fig. 3.13). If a third resistor is joined with the series combination of the two (Fig. 3.14), then all three are said to be in series. Clearly, we can extend this definition to series combination of any number of resistors.



FIGURE 3.14 A series combination of three resistors R_1 , R_2 , R_3 .

Two or more resistors are said to be in *parallel* if one end of all the resistors is joined together and similarly the other ends joined together (Fig. 3.15).

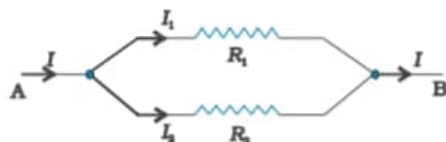


FIGURE 3.15 Two resistors R_1 and R_2 connected in parallel.

Consider two resistors R_1 and R_2 in series. The charge which leaves R_1 must be entering R_2 . Since current measures the rate of flow of charge, this means that the same current I flows through R_1 and R_2 . By Ohm's law:

$$\text{Potential difference across } R_1 = V_1 = IR_1, \text{ and}$$

$$\text{Potential difference across } R_2 = V_2 = IR_2.$$

The potential difference V across the combination is $V_1 + V_2$. Hence,

$$V = V_1 + V_2 = I(R_1 + R_2) \quad (3.36)$$

This is as if the combination had an equivalent resistance R_{eq} , which by Ohm's law is

$$R_{eq} \equiv \frac{V}{I} = (R_1 + R_2) \quad (3.37)$$

If we had three resistors connected in series, then similarly

$$V = IR_1 + IR_2 + IR_3 = I(R_1 + R_2 + R_3). \quad (3.38)$$

This obviously can be extended to a series combination of any number n of resistors R_1, R_2, \dots, R_n . The equivalent resistance R_{eq} is

$$R_{eq} = R_1 + R_2 + \dots + R_n \quad (3.39)$$

Consider now the parallel combination of two resistors (Fig. 3.15). The charge that flows in at A from the left flows out partly through R_1 and partly through R_2 . The currents I, I_1, I_2 shown in the figure are the rates of flow of charge at the points indicated. Hence,

$$I = I_1 + I_2 \quad (3.40)$$

The potential difference between A and B is given by the Ohm's law applied to R_1

$$V = I_1 R_1 \quad (3.41)$$

Also, Ohm's law applied to R_2 gives

$$V = I_2 R_2 \quad (3.42)$$

$$\therefore I = I_1 + I_2 = \frac{V}{R_1} + \frac{V}{R_2} = V \left(\frac{1}{R_1} + \frac{1}{R_2} \right) \quad (3.43)$$

If the combination was replaced by an equivalent resistance R_{eq} , we would have, by Ohm's law

$$I = \frac{V}{R_{eq}} \quad (3.44)$$

Hence,

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} \quad (3.45)$$

We can easily see how this extends to three resistors in parallel (Fig. 3.16).

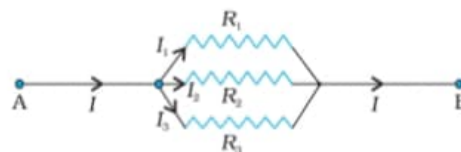


FIGURE 3.16 Parallel combination of three resistors R_1, R_2 and R_3 .

Exactly as before

$$I = I_1 + I_2 + I_3 \quad (3.46)$$

and applying Ohm's law to R_1 , R_2 and R_3 we get,

$$V = I_1 R_1, \quad V = I_2 R_2, \quad V = I_3 R_3 \quad (3.47)$$

So that

$$I = I_1 + I_2 + I_3 = V \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right) \quad (3.48)$$

An equivalent resistance R_{eq} that replaces the combination, would be such that

$$I = \frac{V}{R_{eq}} \quad (3.49)$$

and hence

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \quad (3.50)$$

We can reason similarly for any number of resistors in parallel. The equivalent resistance of n resistors R_1, R_2, \dots, R_n is

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_n} \quad (3.51)$$

These formulae for equivalent resistances can be used to find out currents and voltages in more complicated circuits. Consider for example, the circuit in Fig. (3.17), where there are three resistors R_1, R_2 and R_3 . R_2 and R_3 are in parallel and hence we can replace them by an equivalent R_{eq}^{23} between point B and C with

$$\frac{1}{R_{eq}^{23}} = \frac{1}{R_2} + \frac{1}{R_3}$$

or, $R_{eq}^{23} = \frac{R_2 R_3}{R_2 + R_3} \quad (3.52)$

The circuit now has R_1 and R_{eq}^{23} in series and hence their combination can be replaced by an equivalent resistance with

$$R_{eq}^{123} = R_{eq}^{23} + R_1 \quad (3.53)$$

If the voltage between A and C is V , the current I is given by

$$\begin{aligned} I &= \frac{V}{R_{eq}^{123}} = \frac{V}{R_1 + \left[\frac{R_2 R_3}{R_2 + R_3} \right]} \\ &= \frac{V(R_2 + R_3)}{R_1 R_2 + R_1 R_3 + R_2 R_3} \end{aligned} \quad (3.54)$$

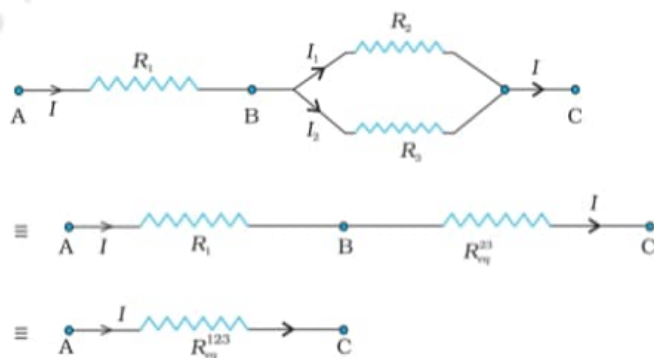


FIGURE 3.17 A combination of three resistors R_1, R_2 and R_3 . R_2, R_3 are in parallel with an equivalent resistance R_{eq}^{23} . R_1 and R_{eq}^{23} are in series with an equivalent resistance R_{eq}^{123} .

Example 3.5 A network of resistors is connected to a 16 V battery with internal resistance of 1Ω as shown in Fig. 3.19: (a) Compute the equivalent resistance of the network. (b) Obtain the current in each resistor. (c) Obtain the voltage drops V_{AB} , V_{BC} and V_{CD} .

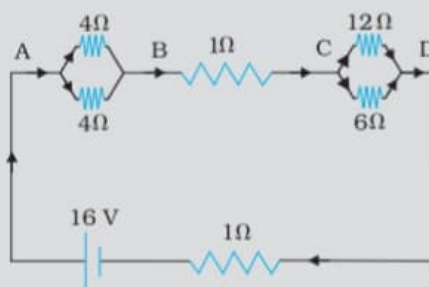


FIGURE 3.19

Solution

(a) The network is a simple series and parallel combination of resistors. First the two 4Ω resistors in parallel are equivalent to a resistor = $[(4 \times 4)/(4 + 4)]\Omega = 2\Omega$. In the same way, the 12Ω and 6Ω resistors in parallel are equivalent to a resistor of $[(12 \times 6)/(12 + 6)]\Omega = 4\Omega$. The equivalent resistance R of the network is obtained by combining these resistors (2Ω and 4Ω) with 1Ω in series, that is,

$$R = 2\Omega + 4\Omega + 1\Omega = 7\Omega$$

(b) The total current I in the circuit is

$$I = \frac{\epsilon}{R+r} = \frac{16V}{(7+1)\Omega} = 2A$$

Consider the resistors between A and B. If I_1 is the current in one of the 4Ω resistors and I_2 the current in the other,

$$I_1 \times 4 = I_2 \times 4$$

that is, $I_1 = I_2$, which is otherwise obvious from the symmetry of the two arms. But $I_1 + I_2 = I = 2A$. Thus,

$$I_1 = I_2 = 1A$$

that is, current in each 4Ω resistor is $1A$. Current in 1Ω resistor between B and C would be $2A$.

Now, consider the resistances between C and D. If I_3 is the current in the 12Ω resistor, and I_4 in the 6Ω resistor,

$$I_3 \times 12 = I_4 \times 6, \text{ i.e., } I_4 = 2I_3$$

$$\text{But, } I_3 + I_4 = I = 2A$$

$$\text{Thus, } I_3 = \left(\frac{2}{3}\right)A, I_4 = \left(\frac{4}{3}\right)A$$

that is, the current in the 12Ω resistor is $(2/3)A$, while the current in the 6Ω resistor is $(4/3)A$.

(c) The voltage drop across AB is

$$V_{AB} = I_1 \times 4 = 1A \times 4\Omega = 4V,$$

This can also be obtained by multiplying the total current between A and B by the equivalent resistance between A and B, that is,

EXAMPLE 3.5

Current Electricity

$$V_{AB} = 2 \text{ A} \times 2 \ \Omega = 4 \text{ V}$$

The voltage drop across BC is

$$V_{BC} = 2 \text{ A} \times 1 \ \Omega = 2 \text{ V}$$

Finally, the voltage drop across CD is

$$V_{CD} = 12 \ \Omega \times I_3 = 12 \ \Omega \times \left(\frac{2}{3}\right) \text{ A} = 8 \text{ V}.$$

This can alternately be obtained by multiplying total current between C and D by the equivalent resistance between C and D, that is,

$$V_{CD} = 2 \text{ A} \times 4 \ \Omega = 8 \text{ V}$$

Note that the total voltage drop across AD is $4 \text{ V} + 2 \text{ V} + 8 \text{ V} = 14 \text{ V}$. Thus, the terminal voltage of the battery is 14 V , while its emf is 16 V . The loss of the voltage ($= 2 \text{ V}$) is accounted for by the internal resistance $1 \ \Omega$ of the battery [$2 \text{ A} \times 1 \ \Omega = 2 \text{ V}$].