**7.31** In the $LCR$ circuit shown in Fig 7.7, the ac driving voltage is $v = v_m \sin \omega t$.

(i) Write down the equation of motion for $q(t)$.

(ii) At $t = t_0$, the voltage source stops and $R$ is short circuited. Now write down how much energy is stored in each of $L$ and $C$.

(iii) Describe subsequent motion of charges.

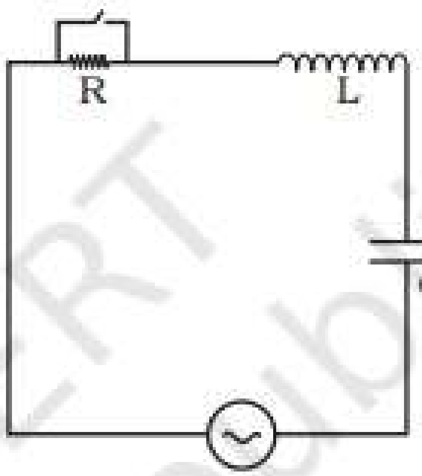
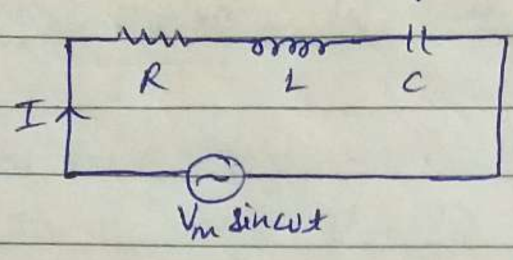



Fig. 7.7

31. In the given series LCR circuit, let 'I' be the current in circuit. Applying Kirchoff's voltage law;




(i) $\therefore$ $V_R + V_L + V_C = V_m \sin(\omega t)$

$$\Rightarrow IR + L\frac{d(I)}{dt} + \frac{q(t)}{C} - V_m \sin(\omega t) = 0 \quad \text{— (1)}$$

As charge changes with time in the circuit;

$$I = \frac{d[q(t)]}{dt}$$

$$\Rightarrow \frac{d(I)}{dt} = \frac{d^2[q(t)]}{dt^2} \quad \text{(Differentiating again)}$$

From (1)

$$R\frac{d[q(t)]}{dt} + L\frac{d^2[q(t)]}{dt^2} + \frac{q(t)}{C} = V_m \sin(\omega t)$$

$$\Rightarrow L\frac{d^2[q(t)]}{dt^2} + R\frac{d[q(t)]}{dt} + \frac{q(t)}{C} = V_m \sin(\omega t)$$

This is the equation for variation of motion of charge w.r.t. time.

(ii) Let time dependent charge in circuit is at phase angle $\phi$ with voltage. Then $q = q_m \cos(\omega t + \phi)$

$$I = \frac{dq}{dt} = \omega q_m \sin(\omega t + \phi) \quad \text{— (2)}$$

$$I_m = \frac{V_m}{Z} = \frac{V_m}{\sqrt{R^2 + (X_C - X_L)^2}} \quad \text{— (3)}$$

$$\tan\phi = \frac{X_C - X_L}{R}$$

At $t = t_0$, R is short circuited, then energy is only stored in L and C, when K is is closed

$$U_L = \frac{1}{2} L I^2 \quad — (4)$$

At $t = t_0$

$$I = I_m \sin(\omega t + \phi) \quad — (5)$$

From (3)

$$I = \frac{V_m}{\sqrt{R^2 + (X_C - X_L)^2}} \sin(\omega t + \phi) \quad — (6)$$

$$\therefore U_L = \frac{1}{2}\left[\frac{V_m}{\sqrt{R^2 + (X_C - X_L)^2}}\right]^2 \sin^2(\omega t + \phi)$$

$$\therefore U_C = \frac{q^2}{2C} = \frac{1}{2C}\left[q_m^2 \cos^2(\omega t + \phi)\right]$$

On comparing (2) & (5)

$$I_m = q_m \omega$$

$$\Rightarrow q_m = \frac{I_m}{\omega}$$

$$\therefore U_C = \frac{1}{2C\omega^2} i_m^2 \cos^2(\omega t + \phi)$$

Using equation (3)

$$U_C = \frac{1}{2C\omega^2}\left[\frac{V_m^2}{R^2 + (X_C - X_L)^2}\right]^2 \cos^2(\omega t + \phi)$$

(ii) When R is short circuited, then, it becomes L-C oscillator. Capacitor will discharge and all energy transfer to L, and back and forth. Hence, there is energy oscillation from electrostatic to magnetic and vice-versa.