Previous Year Questions

QD: Let A & B denote the statements

A: $\cos\alpha + \cos\beta + \cos\gamma = 0$

B: $\sin\alpha + \sin\beta + \sin\gamma = 0$

If $\cos(\beta-\gamma) + \cos(\gamma-\alpha) + \cos(\alpha-\beta) = -\frac{3}{2}$,

then :

1) Both A & B are true 2) Both A & B are false

3) A is true & B is false 4) A is false & B is true

Ans 1)

$\cos(\beta-\gamma) + \cos(\gamma-\alpha) + \cos(\alpha-\beta) = -\frac{3}{2}$

$\Rightarrow 2\cos(\beta-\gamma) + 2\cos(\gamma-\alpha) + 2\cos(\alpha-\beta) = -3$

$\Rightarrow 1+1+1 + 2(\cos\beta\cos\gamma + \sin\beta\sin\gamma) + 2(\cos\gamma\cos\alpha + \sin\gamma\sin\alpha) + 2(\cos\alpha\cos\beta + \sin\alpha\sin\beta) = 0$

$\Rightarrow (\sin^2\alpha + \cos^2\alpha) + (\sin^2\beta + \cos^2\beta) + (\sin^2\gamma + \cos^2\gamma) + 2\cos\alpha\cos\beta + 2\cos\beta\cos\gamma + 2\cos\gamma\cos\alpha + 2\sin\alpha\sin\beta + 2\sin\beta\sin\gamma + 2\sin\gamma\sin\alpha = 0$

$\Rightarrow (\sin^2\alpha + \sin^2\beta + \sin^2\gamma + 2\sin\alpha\sin\beta + 2\sin\beta\sin\gamma + 2\sin\gamma\sin\alpha) + (\cos^2\alpha + \cos^2\beta + \cos^2\gamma + 2\cos\alpha\cos\beta + 2\cos\beta\cos\gamma + 2\cos\gamma\cos\alpha) = 0$

$\Rightarrow (\sin\alpha + \sin\beta + \sin\gamma)^2 + (\cos\alpha + \cos\beta + \cos\gamma)^2 = 0$

Only possible when,

$\sin\alpha + \sin\beta + \sin\gamma = 0$ & $\cos\alpha + \cos\beta + \cos\gamma = 0$

∴ Both A & B are true.

Q2) The possible value of $\theta \in (0, \pi)$ such that $\sin(\theta) + \sin(4\theta) + \sin(7\theta) = 0$ are

1) $\frac{2\pi}{9}, \frac{\pi}{4}, \frac{4\pi}{9}, \frac{\pi}{2}, \frac{3\pi}{4}, \frac{8\pi}{9}$

2) $\frac{\pi}{4}, \frac{5\pi}{12}, \frac{\pi}{2}, \frac{2\pi}{3}, \frac{3\pi}{4}, \frac{8\pi}{9}$

3) $\frac{2\pi}{9}, \frac{\pi}{4}, \frac{\pi}{2}, \frac{2\pi}{3}, \frac{3\pi}{4}, \frac{35\pi}{36}$

4) $\frac{2\pi}{9}, \frac{\pi}{4}, \frac{\pi}{2}, \frac{2\pi}{3}, \frac{3\pi}{4}, \frac{8\pi}{9}$

Ans 2) $\sin\theta + \sin 4\theta + \sin 7\theta = 0$

$\Rightarrow 2\sin\left(\frac{\theta + 7\theta}{2}\right)\cos\left(\frac{7\theta - \theta}{2}\right) + \sin 4\theta = 0$

$\Rightarrow 2\sin 4\theta \cos 3\theta + \sin 4\theta = 0$

$\Rightarrow \sin 4\theta (2\cos 3\theta + 1) = 0$

$\sin 4\theta = 0$

$\Rightarrow \theta = 0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \pi$ but $0 \& \pi$ are not included.

$2\cos 3\theta + 1 = 0$

$\Rightarrow \cos 3\theta = \frac{-1}{2}$

$\Rightarrow 3\theta = \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{8\pi}{3}, \frac{10\pi}{3}$

$\Rightarrow \theta = \frac{2\pi}{9}, \frac{4\pi}{9}, \frac{8\pi}{9}, \frac{10\pi}{9}$ ~~but~~ but $\frac{10\pi}{9} \notin (0, \pi)$

$\therefore \theta = \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \frac{2\pi}{9}, \frac{4\pi}{9}, \frac{8\pi}{9}$