

Problems

Q1) Find the value of $\sqrt{3} \operatorname{cosec} 20^\circ - \sec 20^\circ$

Ans)
$$= \frac{\sqrt{3}}{\sin 20^\circ} - \frac{1}{\cos 20^\circ}$$

$$= \frac{\sqrt{3} \cos 20^\circ - \sin 20^\circ}{\sin 20^\circ \cos 20^\circ}$$

$$= 4 \left(\frac{\sin 60^\circ \cos 20^\circ - \cos 60^\circ \sin 20^\circ}{\sin 40^\circ} \right)$$

$$= 4 \left(\frac{\sin(60^\circ - 20^\circ)}{\sin 40^\circ} \right) = 4$$

Q2) Show that $\tan 3x \tan 2x \tan x = \tan 3x - \tan 2x - \tan x$

Ans) Let $3x = 2x + x$

$$\tan 3x = \tan(2x + x)$$

$$\Rightarrow \tan 3x = \frac{\tan 2x + \tan x}{1 - \tan 2x \tan x}$$

$$1 - \tan 2x \tan x$$

$$\Rightarrow \tan 3x - \tan 3x \tan 2x \tan x = \tan 2x + \tan x$$

$$\Rightarrow \tan 3x \tan 2x \tan x = \tan 3x - \tan 2x - \tan x$$

Q3) Prove that:

$$\cos^2 x + \cos^2 \left(x + \frac{\pi}{3} \right) + \cos^2 \left(x - \frac{\pi}{3} \right) = \frac{3}{2}$$

Ans) LHS = $\cos^2 x + \left(\frac{\cos x \cos \frac{\pi}{3} - \sin x \sin \frac{\pi}{3}}{2} \right)^2 + \left(\frac{\cos x \cos \frac{\pi}{3} + \sin x \sin \frac{\pi}{3}}{2} \right)^2$

$$= \cos^2 x + \left(\frac{1}{2} \cos x - \frac{\sqrt{3}}{2} \sin x \right)^2 + \left(\frac{1}{2} \cos x + \frac{\sqrt{3}}{2} \sin x \right)^2$$

$$= \cos^2 x + \frac{1}{2} \cos^2 x + \frac{3}{2} \sin^2 x = \frac{3}{2} \cos^2 x + \frac{3}{2} \sin^2 x = \frac{3}{2} (\cos^2 x + \sin^2 x) = \frac{3}{2} = \text{RHS}$$

Q4

$$\begin{aligned} \text{Q4)} \quad & \text{Find the value of } \cos 570^\circ \sin 510^\circ + \sin(-330^\circ) \cos(-390^\circ) \\ & = \cos(570^\circ) \sin(510^\circ) - \sin(330^\circ) \cos(390^\circ) \quad (\because \sin(-x) = -\sin x) \\ & = \cos(210^\circ) \sin(150^\circ) - \sin(330^\circ) \cos(30^\circ) \end{aligned}$$

$$(\because \sin(2\pi+x) = \sin x \text{ \& } \cos(2\pi+x) = \cos x)$$

$$= -\cos(30^\circ) \cos(60^\circ) + \sin(30^\circ) \cos(30^\circ)$$

$$= -\frac{\sqrt{3}}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{\sqrt{3}}{2}$$

$$= 0$$

$$\text{Q5)} \quad \text{Now that } 2\sin^2\beta + 4\cos(x+\beta) \sin x \sin\beta + \cos[2(x+\beta)] = \cos(2x)$$

$$\text{Q5)} \quad \text{LHS} = 2\sin^2\beta + 4\cos(x+\beta) \sin x \sin\beta + \cos 2(x+\beta)$$

$$= 2\sin^2\beta + 4(\cos x \cos\beta - \sin x \sin\beta) \sin x \sin\beta$$

$$+ (\cos 2x \cos 2\beta - \sin 2x \sin 2\beta)$$

$$= 2\sin^2\beta + 4\sin x \cos x \sin\beta \cos\beta - 4\sin^2 x \sin^2\beta + \cos 2x \cos 2\beta$$

$$- \sin 2x \sin 2\beta$$

$$= 2\sin^2\beta + \cancel{\sin 2x \sin 2\beta} - 4\sin^2 x \sin^2\beta + \cos 2x \cos 2\beta$$

$$- \cancel{\sin 2x \sin 2\beta}$$

$$= \cancel{2\sin^2\beta} (1 - \cos 2\beta) - (2\sin^2 x)(2\sin^2\beta) + \cos 2x \cos 2\beta$$

$$= (1 - \cos 2\beta) - (1 - \cos 2x)(1 - \cos 2\beta) + \cos 2x \cos 2\beta$$

$$\therefore = (1 - \cos 2\beta)(1 - 1 + \cos 2x) + \cos 2x \cos 2\beta$$

$$= \cos 2x$$

$$= \text{RHS}$$