Ques: The integral $\int \frac{\sec^2 x}{(\sec x+\tan x)^{9/2}}dx$ equals (for some arbitrary constant K)

(A) $-\frac{1}{(\sec x+\tan x)^{11/2}}\left[\frac{1}{11}-\frac{1}{7}(\sec x+\tan x)^2\right]+K$

(B) $\frac{1}{(\sec x+\tan x)^{11/2}}\left\{\frac{1}{11}-\frac{1}{7}(\sec x+\tan x)^2\right\}+K$

(C) $-\frac{1}{(\sec x+\tan x)^{11/2}}\left\{\frac{1}{11}+\frac{1}{7}(\sec x+\tan x)^2\right\}+K$

(D) $\frac{1}{(\sec x+\tan x)^{11/2}}\left\{\frac{1}{11}+\frac{1}{7}(\sec x+\tan x)^2\right\}+K$

solution:

$$I=\frac{\sec^2 x\, dx}{(\sec x+\tan x)^{9/2}}$$

let $\sec x+\tan x = t$, $\sec x-\tan x=\frac{1}{t}$

$$\Rightarrow 2\sec x = t+\frac{1}{t}$$

$$\sec x = \frac{t+1/t}{2}$$

$$I=\frac{\sec x\cdot\sec x\,(\sec x+\tan x)\,dx}{(\sec x+\tan x)^{11/2}}$$

$$=\frac{1}{2}\int\frac{\left(t+\frac{1}{t}\right)\,dx}{t^{11/2}}$$

$$= \int \frac{\left(t^{-9/2} + t^{-13/2}\right)}{2} dt$$

$$= \frac{1}{2}\left\{ \frac{t^{-7/2}}{-7/2} + \frac{t^{-11/2}}{-11/2} \right] + c$$

$$= - \frac{1}{t^{11/2}} \left[\frac{1}{11} + \frac{1}{7} t^2 \right] + c$$

$$= - \frac{1}{(\sec x + \tan x)^{11/2}} \left[\frac{1}{11} + \frac{1}{7} (\sec x + \tan x) \right] + c$$

Ques: let $f: R \to R$ and $g: R \to R$ be two non-constant differentiable functions. if $f'(x) = e^{(f(x) - g(x))} g'(x)$ for all $x \in R$ and $f(1) = g(2) = 1$ then which of the following statements are true?

(A) $f(2) < 1 - \log_e 2$

(B) $f(2) > 1 - \log_e 2$

(C) $g(1) > 1 - \log_e 2$

(D) $g(1) < 1 - \log_e 2$

Solution:-

given:- $f'(x) = e^{(f(x) - g(x))} g'(x) \ \forall x \in R$

$$\Rightarrow e^{-f(x)} \cdot f'(x) - e^{-g(x)} g'(x) = 0$$

$$\Rightarrow \int \left(e^{-f(x)} f'(x) - e^{-g(x)} \cdot g'(x)\right) dx = c$$

$$\Rightarrow -e^{-f(x)} + e^{-g(x)} = c$$

$$\Rightarrow -e^{-f(1)} + e^{-g(1)} = -e^{-f(2)} + e^{-g(2)}$$

$$\Rightarrow -\frac{1}{e} + e^{-g(1)} = -e^{-f(2)} + \frac{1}{e}$$

$$\Rightarrow e^{-f(2)} + e^{-g(1)} = \frac{2}{e}$$

$$\therefore \quad e^{-f(2)} < \frac{2}{e} \quad \text{and} \quad e^{-g(1)} < \frac{2}{e}$$

$$\Rightarrow -f(2) < \ln 2 - 1 \quad \text{and} \quad -g(1) < \ln 2 - 1$$

$$f(2) > 1 - \ln 2 \quad \text{and} \quad g(1) > 1 - \ln 2$$