① Conditional Probability

If A & B are any events in S, then the conditional ~~probabi~~ probability of B relative to A i.e. probability of occurrence of B when A has occurred, is given by,

$$P(B/A) = \frac{P(B\cap A)}{P(A)}, \quad P(A) \neq 0$$

* special case (when events A & B independent)

$$\cancel{P(A \cap B)} \quad P(B\cap A) = P(A)\cdot P(B)$$

$$P(B|A) = \frac{P(B\cap A)}{P(A)} = \frac{\cancel{P(A)}\,P(B)}{\cancel{P(A)}} = P(B)$$

② Baye's Theorem (or Inverse Probability)

Let $A_1, A_2, \dots, A_n$ be n mutually exclusive & exhaustive events of the sample space S & A is event which can occur with any of the events

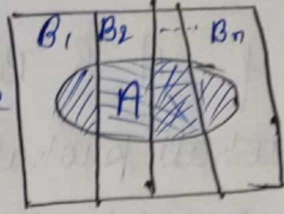
then,
$$P(A_i/A) = \frac{P(A_i)\,P(A/A_i)}{\sum_{i=1}^{n} P(A_i)\,P(A|A_i)}$$

③ If A & B are mutually exclusive, then

$P(A\cap B) = 0$,

$$P(A\cup B) = P(A) + P(B)$$

(4) Total Probability Theorem



Let an event A of an experiment occurs with its n mutually exclusive & exhaustive events $B_1, B_2, B_3 \ldots, B_n$

then, total probability of occurrence of events A

$$P(A) = P(AB_1) + P(AB_2) + \ldots + P(AB_n)$$

$$= \sum_{i=1}^{n} P(AB_i)$$

$$P(A) = P(B_1)P(A|B_1) + P(B_2)P(A|B_2) + \cdots + P(B_n)P(A|B_n)$$

$$= \sum_{i=1}^{n} P(B_i)P(A|B_i)$$

(5) Some Important results

(a) Let A & B be two events, then

(i) $P(A) + P(\bar{A}) = 1$

(ii) $P(A+B) = 1 - P(\bar{A}\bar{B})$

(iii) $P(A|B) = \dfrac{P(AB)}{P(B)}$

(iv) $P(A+B) = P(AB) + P(\bar{A}B) + P(A\bar{B})$

(v) $A \subset B \Rightarrow P(A) \leq P(B)$

(vi) $P(\bar{A}B) = P(B) - P(AB)$

(vii) $P(AB) \leq P(A)P(B) \leq P(A+B) \leq P(A) + P(B)$

(viii) $P(AB) = P(A) + P(B) - P(A+B)$

(ix) $P(\text{Exactly one event}) = P(A\bar{B}) + P(\bar{A}B)$

$$= P(A) + P(B) - 2P(AB) = P(A+B) - P(AB)$$

(x) $P(\text{neither } A \text{ nor } B) = P(\bar{A}\bar{B}) = 1 - P(A+B)$

(xi) $P(\bar{A} + \bar{B}) = 1 - P(AB)$

(b) Number of exhaustive cases of tossing n coins simultaneously (or of tossing a coin n times)
$= 2^n$

(c) Number of exhaustive cases of throwing n dice simultaneously (or throwing one dice n times)
$= 6^n$

(d) Playing cards

(i) Total cards : 52 (26 red, 26 black)

(ii) Four suits : Heart, Diamond, Spade, Club – 13 cards each

(iii) Court cards : 12 (4 kings, 4 queens, 4 jacks)

(iv) Honour cards : 16 (4 aces, 4 kings, 4 queens, 4 jacks)

(e) Probability regarding n letters & their envelops:-
If n letters are placed into n directed envelops at random, then

(i) Probability that all letters are in right envelopes $= 1/n!$

(ii) Probability that all letters are not in right envelops $= 1 - 1/n!$

(iii) Probability that no letters is in right envelopes $= \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \ldots + (-1)^n \frac{1}{n!}$

(iv) Probability that exactly r letters are in right envelopes

$$= \frac{1}{r!}\left(\frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \cdots + (-1)^{n-r}\frac{1}{(n-r)!}\right)$$