3.5.1 Mobility

As we have seen, conductivity arises from mobile charge carriers. In metals, these mobile charge carriers are electrons; in an ionised gas, they are electrons and positive charged ions; in an electrolyte, these can be both positive and negative ions.

An important quantity is the *mobility* μ defined as the magnitude of the drift velocity per unit electric field:

$$\mu = \frac{|\mathbf{v}_d|}{E} \tag{3.24}$$

The SI unit of mobility is m^2/Vs and is 10^4 of the mobility in practical units (cm^2/Vs). Mobility is positive. From Eq. (3.17), we have

$$v_d = \frac{e\tau E}{m}$$

(3.25)

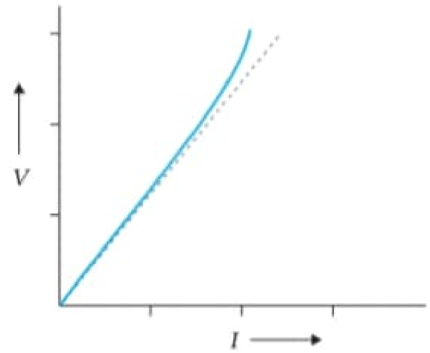

FIGURE 3.5 The dashed line represents the linear Ohm's law. The solid line is the voltage *V* versus current *I* for a good conductor.

3.6 Limitations of Ohm's Law

Although Ohm's law has been found valid over a large class of materials, there do exist materials and devices used in electric circuits where the proportionality of *V* and *I* does not hold. The deviations broadly are one or more of the following types:

(a) *V* ceases to be proportional to *I* (Fig. 3.5).

(b) The relation between *V* and *I* depends on the sign of *V*. In other words, if *I* is the current for a certain *V*, then reversing the direction of *V* keeping its magnitude fixed, does not produce a current of the same magnitude as *I* in the opposite direction (Fig. 3.6). This happens, for example, in a diode which we will study in Chapter 14.

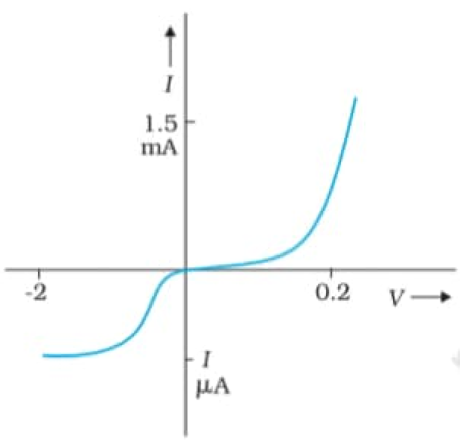

FIGURE 3.6 Characteristic curve of a diode. Note the different scales for negative and positive values of the voltage and current.

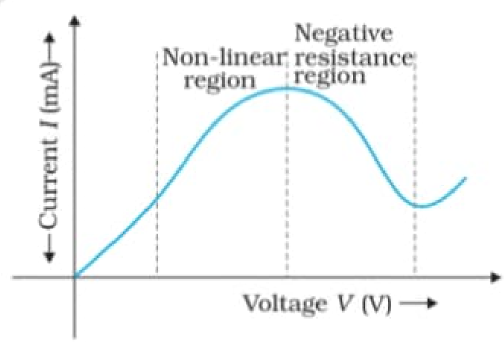

FIGURE 3.7 Variation of current versus voltage for GaAs.

(c) The relation between *V* and *I* is not unique, i.e., there is more than one value of *V* for the same current *I* (Fig. 3.7). A material exhibiting such behaviour is GaAs.

Materials and devices not obeying Ohm's law in the form of Eq. (3.3) are actually widely used in electronic circuits. In this and a few subsequent chapters, however, we will study the electrical currents in materials that obey Ohm's law.

Resistors in the higher range are made mostly from carbon. Carbon resistors are compact, inexpensive and thus find extensive use in electronic circuits. Carbon resistors are small in size and hence their values are given using a colour code.

Table 3.2 Resistor colour codes

Colour	Number	Multiplier	Tolerance (%)
Black	0	1	
Brown	1	10^1	
Red	2	10^2	
Orange	3	10^3	
Yellow	4	10^4	
Green	5	10^5	
Blue	6	10^6	
Violet	7	10^7	
Gray	8	10^8	
White	9	10^9	
Gold		10^{-1}	5
Silver		10^{-2}	10
No colour			20

The resistors have a set of co-axial coloured rings on them whose significance are listed in Table 3.2. The first two *bands* from the end indicate the first two significant figures of the resistance in ohms. The third band indicates the decimal multiplier (as listed in Table 3.2). The last band stands for tolerance or possible variation in percentage about the indicated values. Sometimes, this last band is absent and that indicates a tolerance of 20% (Fig. 3.8). For example, if the four colours are orange, blue, yellow and gold, the resistance value is $36 \times 10^4\ \Omega$, with a tolerence value of 5%.

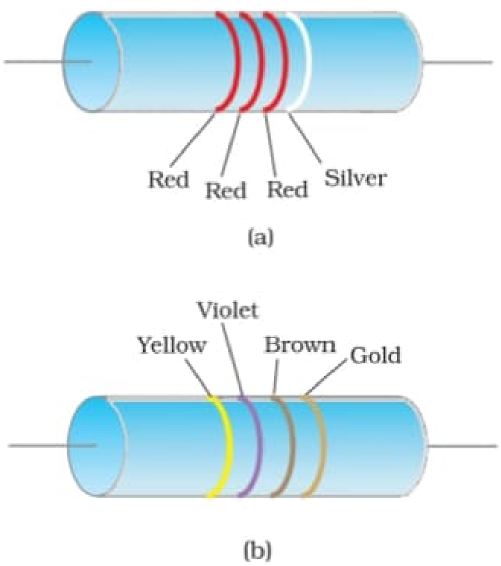

FIGURE 3.8 Colour coded resistors (a) $(22 \times 10^2\ \Omega) \pm 10\%$, (b) $(47 \times 10\ \Omega) \pm 5\%$.

3.8 Temperature Dependence of Resistivity

The resistivity of a material is found to be dependent on the temperature. Different materials do not exhibit the same dependence on temperatures. Over a limited range of temperatures, that is not too large, the resistivity of a metallic conductor is approximately given by,

$$\rho_T = \rho_0\,[1 + \alpha\,(T - T_0)] \qquad (3.26)$$

where ρ_T is the resistivity at a temperature T and ρ_0 is the same at a reference temperature T_0. α is called the *temperature co-efficient of resistivity*, and from Eq. (3.26), the dimension of α is (Temperature)$^{-1}$.

For metals, α is positive and values of α for some metals at $T_0 = 0°C$ are listed in Table 3.1.

The relation of Eq. (3.26) implies that a graph of ρ_T plotted against T would be a straight line. At temperatures much lower than 0°C, the graph, however, deviates considerably from a straight line (Fig. 3.9).

Equation (3.26) thus, can be used approximately over a limited range of T around any reference temperature T_0, where the graph can be approximated as a straight line.

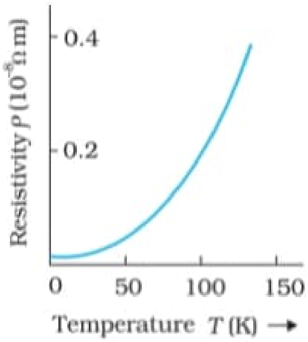

FIGURE 3.9 Resistivity ρ_T of copper as a function of temperature T.

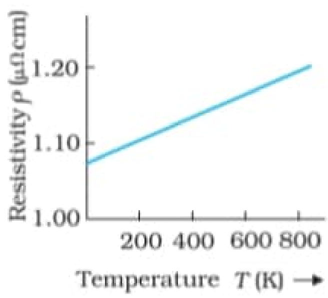

FIGURE 3.10 Resistivity ρ_T of nichrome as a function of absolute temperature T.

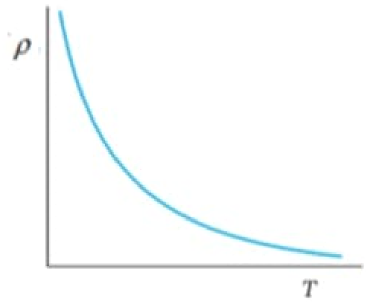

FIGURE 3.11 Temperature dependence of resistivity for a typical semiconductor.

Some materials like Nichrome (which is an alloy of nickel, iron and chromium) exhibit a very weak dependence of resistivity with temperature (Fig. 3.10). Manganin and constantan have similar properties. These materials are thus widely used in wire bound standard resistors since their resistance values would change very little with temperatures.

Unlike metals, the resistivities of semiconductors decrease with increasing temperatures. A typical dependence is shown in Fig. 3.11.

We can qualitatively understand the temperature dependence of resistivity, in the light of our derivation of Eq. (3.23). From this equation, resistivity of a material is given by

$$\rho = \frac{1}{\sigma} = \frac{m}{n e^2 \tau} \qquad (3.27)$$

ρ thus depends inversely both on the number n of free electrons per unit volume and on the average time τ between collisions. As we increase temperature, average speed of the electrons, which act as the carriers of current, increases resulting in more frequent collisions. The average time of collisions τ, thus decreases with temperature.

In a metal, n is not dependent on temperature to any appreciable extent and thus the decrease in the value of τ with rise in temperature causes ρ to increase as we have observed.

For insulators and semiconductors, however, n increases with temperature. This increase more than compensates any decrease in τ in Eq.(3.23) so that for such materials, ρ decreases with temperature.

Example 3.3 An electric toaster uses nichrome for its heating element. When a negligibly small current passes through it, its resistance at room temperature (27.0 °C) is found to be 75.3 Ω. When the toaster is connected to a 230 V supply, the current settles, after a few seconds, to a steady value of 2.68 A. What is the steady temperature of the nichrome element? The temperature coefficient of resistance of nichrome averaged over the temperature range involved, is 1.70×10^{-4} °C^{-1}.

Solution When the current through the element is very small, heating effects can be ignored and the temperature T_1 of the element is the same as room temperature. When the toaster is connected to the supply, its initial current will be slightly higher than its steady value of 2.68 A. But due to heating effect of the current, the temperature will rise. This will cause an increase in resistance and a slight decrease in current. In a few seconds, a steady state will be reached when temperature will rise no further, and both the resistance of the element and the current drawn will achieve steady values. The resistance R_2 at the steady temperature T_2 is

$$R_2 = \frac{230\text{ V}}{2.68\text{ A}} = 85.8\ \Omega$$

Using the relation

$$R_2 = R_1\ [1 + \alpha\ (T_2 - T_1)]$$

with $\alpha = 1.70 \times 10^{-4}$ °C^{-1}, we get

$$T_2 - T_1 = \frac{(85.8 - 75.3)}{(75.3) \times 1.70 \times 10^{-4}} = 820\ °\text{C}$$

that is, $T_2 = (820 + 27.0)$ °C = 847 °C

Thus, the steady temperature of the heating element (when heating effect due to the current equals heat loss to the surroundings) is 847 °C.

EXAMPLE 3.3

Example 3.4 The resistance of the platinum wire of a platinum resistance thermometer at the ice point is 5 Ω and at steam point is 5.23 Ω. When the thermometer is inserted in a hot bath, the resistance of the platinum wire is 5.795 Ω. Calculate the temperature of the bath.

Solution $R_0 = 5\ \Omega$, $R_{100} = 5.23\ \Omega$ and $R_t = 5.795\ \Omega$

Now, $$t = \frac{R_t - R_0}{R_{100} - R_0} \times 100, \qquad R_t = R_0\,(1 + \alpha t)$$

$$= \frac{5.795 - 5}{5.23 - 5} \times 100$$

$$= \frac{0.795}{0.23} \times 100 = 345.65\ °\text{C}$$

EXAMPLE 3.4