Ques:- A doctor is to visit a patient. From the past experiences, it is known that the probabilities that he will come by train, bus, scooter or by other means of transport are respectively $\frac{3}{10}, \frac{1}{5}, \frac{1}{10}$ and $\frac{2}{5}$. The probabilities that he will be late are $\frac{1}{4}, \frac{1}{3}$ and $\frac{1}{12}$, if he comes by train, bus, scooter respectively.

but if he comes by other means of transport then he will not be late. when he arrives, he is late. what is the probability that he comes by train?

Soln:- let E → Event when doctor comes late.

D_1, D_2, D_3, D_4 are the events when he comes by train, bus, scooter and other means of transport respectively.

$P(D_1) = \frac{3}{10}, \quad P(D_2) = \frac{1}{5}, \quad P(D_3) = \frac{1}{10}$

$P(D_4) = \frac{2}{5}$.

$P(E/D_1) = \frac{1}{4}, \quad P(E/D_2) = \frac{1}{3}$

$P(E/D_3) = \frac{1}{12}, \quad P(E/D_4) = 0.$

and we know from Bayes theorem:-

$$P(D_1|E) = \frac{P(D_1)\,P(E|D_1)}{P(D_1).P(E|D_1) + P(D_2).P(E|D_2) + P(D_3)P(E|D_3) + P(D_4)P(E|D_4)}$$

$$= \frac{\frac{3}{10}\cdot\frac{1}{4}}{\frac{3}{10}\cdot\frac{1}{4} + \frac{1}{5}\cdot\frac{1}{3} + \frac{1}{10}\cdot\frac{1}{12} + \frac{2}{5}\cdot 0} = \frac{1}{2}.$$