Que:- If a fair coin is tossed 10 times, find the probability of
(i) exactly six heads
(ii) atleast six heads
(iii) atmost six heads

Solun:- This is problem of Binomial distribution
let x be the number of heads in n experiments so

$$P(X=x) = {}^{n}C_{x}\, q^{n-x} p^{x} \quad , x = 0,1,2,\ldots n$$

$$n = 10 \,, \quad p = \frac{1}{2} \,, \quad q = 1-p = \frac{1}{2}.$$

Thus,

$$P(X=x) = {}^{10}C_{x} \left(\frac{1}{2}\right)^{10-x} \left(\frac{1}{2}\right)^{x}$$

$$= {}^{10}C_{x} \left(\frac{1}{2}\right)^{10}$$

(i) here $x = 6$

so $P(X=6) = {}^{10}C_{6} \left(\frac{1}{2}\right)^{10}$

$$= \frac{105}{512}.$$

(ii) atleast six heads so $X \geq 6$
but $X \leq 10$

$P(\text{atleast six heads}) = P(X=6) + P(X=7)$
$+ P(X=8) + P(X=9) + P(X=10)$

$= {}^{10}C_6 \left(\frac{1}{2}\right)^{10} + {}^{10}C_7 \left(\frac{1}{2}\right)^{10} + {}^{10}C_8 \left(\frac{1}{2}\right)^{10}$

$+ {}^{10}C_9 \left(\frac{1}{2}\right)^{10} + {}^{10}C_{10} \left(\frac{1}{2}\right)^{10}$

$= \frac{193}{512}$,

(iii) atmost six heads

$P(X \leq 6) = P(X=0) + P(X=1) + P(X=2) + P(X=3)$
$+ P(X=4) + P(X=5) + P(X=6)$

$= \left(\frac{1}{2}\right)^{10} + {}^{10}C_1 \left(\frac{1}{2}\right)^{10} + {}^{10}C_2 \left(\frac{1}{2}\right)^{10} +$

${}^{10}C_3 \left(\frac{1}{2}\right)^{10} + {}^{10}C_4 \left(\frac{1}{2}\right)^{10} + {}^{10}C_5 \left(\frac{1}{2}\right)^{10}$

$+ {}^{10}C_6 \left(\frac{1}{2}\right)^{10}$

$= \frac{53}{64}$.