10. The Cartesian product A × A has 9 elements among which are found (–1, 0) and (0,1). Find the set A and the remaining elements of A × A.

2.3 Relations

Consider the two sets P = {*a*, *b*, *c*} and Q = {Ali, Bhanu, Binoy, Chandra, Divya}.

The cartesian product of P and Q has 15 ordered pairs which can be listed as P × Q = {(*a*, Ali), (*a*,Bhanu), (*a*, Binoy), ..., (*c*, Divya)}.

We can now obtain a subset of P × Q by introducing a relation R between the first element *x* and the second element *y* of each ordered pair (*x*, *y*) as

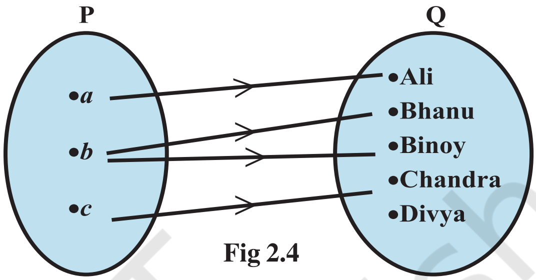

Fig 2.4

R= { (*x*,*y*): *x* is the first letter of the name *y*, $x \in$ P, $y \in$ Q}.

Then R = {(*a*, Ali), (*b*, Bhanu), (*b*, Binoy), (*c*, Chandra)}

A visual representation of this relation R (called an *arrow diagram*) is shown in Fig 2.4.

Definition 2 A relation R from a non-empty set A to a non-empty set B is a subset of the cartesian product A × B. The subset is derived by describing a relationship between the first element and the second element of the ordered pairs in A × B. The second element is called the *image* of the first element.

Definition 3 The set of all first elements of the ordered pairs in a relation R from a set A to a set B is called the *domain* of the relation R.

Definition 4 The set of all second elements in a relation R from a set A to a set B is called the *range* of the relation R. The whole set B is called the *codomain* of the relation R. Note that range $\subset$ codomain.

Remarks (i) A *relation* may be represented algebraically either by the *Roster method* or by the *Set-builder method.*

(ii) An arrow diagram is a visual representation of a relation.

Example 7 Let A = {1, 2, 3, 4, 5, 6}. Define a relation R from A to A by

R = {(*x*, *y*) : *y* = *x* + 1 }

(i) Depict this relation using an arrow diagram.

(ii) Write down the domain, codomain and range of R.

Solution (i) By the definition of the relation,

R = {(1,2), (2,3), (3,4), (4,5), (5,6)}.

The corresponding arrow diagram is shown in Fig 2.5.

(ii) We can see that the domain ={1, 2, 3, 4, 5,}

Similarly, the range = {2, 3, 4, 5, 6} and the codomain = {1, 2, 3, 4, 5, 6}.

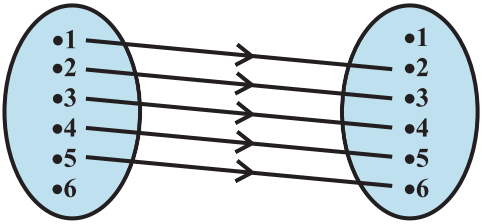

Fig 2.5

Example 8 The Fig 2.6 shows a relation between the sets P and Q. Write this relation (i) in set-builder form, (ii) in roster form. What is its domain and range?

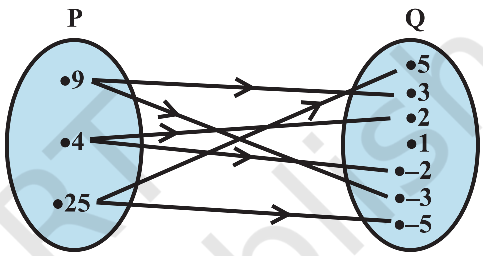

Fig 2.6

Solution It is obvious that the relation R is "*x* is the square of y".

(i) In set-builder form, R = {(*x*, *y*): *x* is the square of *y*, $x \in$ P, $y \in$ **Q**}

(ii) In roster form, R = {(9, 3), (9, –3), (4, 2), (4, –2), (25, 5), (25, –5)}

The domain of this relation is {4, 9, 25}.

The range of this relation is {– 2, 2, –3, 3, –5, 5}.

Note that the element 1 is not related to any element in set P.

The set Q is the codomain of this relation.

Note The total number of relations that can be defined from a set A to a set B is the number of possible subsets of A × B. If n(A) = p and n(B) = q, then n (A × B) = pq and the total number of relations is 2^{pq}.

Example 9 Let A = {1, 2} and B = {3, 4}. Find the number of relations from A to B.

Solution We have,

A × B = {(1, 3), (1, 4), (2, 3), (2, 4)}.

Since n (A×B) = 4, the number of subsets of A×B is 2^4. Therefore, the number of relations from A into B will be 2^4.

Remark A relation R from A to A is also stated as a relation on A.

EXERCISE 2.2

1. Let A = {1, 2, 3,...,14}. Define a relation R from A to A by R = {(*x*, *y*) : 3*x* – *y* = 0, where *x*, $y \in$ A}. Write down its domain, codomain and range.

2. Define a relation R on the set **N** of natural numbers by R = {(*x*, *y*) : *y* = *x* + 5, *x* is a natural number less than 4; *x*, *y* ∈ **N**}. Depict this relationship using roster form. Write down the domain and the range.

3. A = {1, 2, 3, 5} and B = {4, 6, 9}. Define a relation R from A to B by R = {(*x*, *y*): the difference between *x* and *y* is odd; *x* ∈ A, *y* ∈ B}. Write R in roster form.

4. The Fig2.7 shows a relationship between the sets P and Q. Write this relation

(i) in set-builder form (ii) roster form. What is its domain and range?

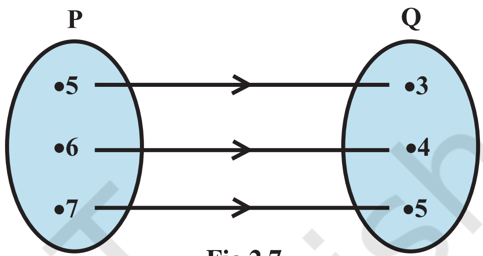

Fig 2.7

5. Let A = {1, 2, 3, 4, 6}. Let R be the relation on A defined by {(*a*, *b*): *a* , *b* ∈ A, *b* is exactly divisible by *a*}.

(i) Write R in roster form

(ii) Find the domain of R

(iii) Find the range of R.

6. Determine the domain and range of the relation R defined by R = {(*x*, *x* + 5) : *x* ∈ {0, 1, 2, 3, 4, 5}}.

7. Write the relation R = {(x, x^3) : *x* is a prime number less than 10} in roster form.

8. Let A = {*x*, *y*, *z*} and B = {1, 2}. Find the number of relations from A to B.

9. Let R be the relation on **Z** defined by R = {(*a*,*b*): *a*, *b* ∈ **Z**, *a* – *b* is an integer}. Find the domain and range of R.

2.4 Functions

In this Section, we study a special type of relation called *function.* It is one of the most important concepts in mathematics. We can, visualise a function as a rule, which produces new elements out of some given elements. There are many terms such as 'map' or 'mapping' used to denote a function.

Definition 5 A relation *f* from a set A to a set B is said to be a *function* if every element of set A has one and only one image in set B.

In other words, a function *f* is a relation from a non-empty set A to a non-empty set B such that the domain of *f* is A and no two distinct ordered pairs in *f* have the same first element.

If *f* is a function from A to B and (*a*, *b*) ∈ *f*, then *f* (*a*) = *b*, where *b* is called the *image* of *a* under *f* and *a* is called the *preimage* of *b* under *f*.