Past Year Questions from Relations

4 JEE Main 2021 (Online) 31st August Morning Shift

MCQ (Single Correct Answer)

Which of the following is not correct for relation R on the set of real numbers ?

A $(x, y) \in R \Leftrightarrow 0 < |x| - |y| \leq 1$ is neither transitive nor symmetric.

B $(x, y) \in R \Leftrightarrow 0 < |x - y| \leq 1$ is symmetric and transitive.

C $(x, y) \in R \Leftrightarrow |x| - |y| \leq 1$ is reflexive but not symmetric.

D $(x, y) \in R \Leftrightarrow |x - y| \leq 1$ is reflexive nd symmetric.

Explanation

Note that (a, b) and (b, c) satisfy $0 < |x - y| \leq 1$ but (a, c) does not satisfy it so $0 \leq |x - y| \leq 1$ is symmetric but not transitive.

For example,

$x = 0.2, y = 0.9, z = 1.5$

$0 \leq |x - y| = 0.7 \leq 1$

$0 \leq |y - z| = 0.6 \leq 1$

But $|x - z| = 1.3 > 1$

So, (b) is correct.

Concept of symmetric relation is used in this question.

4 JEE Main 2021 (Online) 16th March Evening Shift

MCQ (Single Correct Answer)

Let A = {2, 3, 4, 5, ..., 30} and '$\simeq$' be an equivalence relation on A $\times$ A, defined by (a, b) $\simeq$ (c, d), if and only if ad = bc. Then the number of ordered pairs which satisfy this equivalence relation with ordered pair (4, 3) is equal to :

A 5

B 6

C 8

D 7

Explanation

ad = bc

(a, b) R (4, 3) $\Rightarrow$ 3a = 4b

$a = \frac{4}{3}b$

b must be multiple of 3

b = {3, 6, 9 ..... 30}

(a, b) = {(4, 3), (8, 16), (12, 9), (16, 12), (20, 15), (24, 18), (28, 21)}

$\Rightarrow$ 7 ordered pair

1 JEE Main 2020 (Online) 3rd September Evening Slot

MCQ (Single Correct Answer)

Let R_1 and R_2 be two relation defined as follows :

$R_1 = \{(a, b) \in R^2 : a^2 + b^2 \in Q\}$ and

$R_2 = \{(a, b) \in R^2 : a^2 + b^2 \notin Q\}$,

where Q is the set of all rational numbers. Then :

A Neither R_1 nor R_2 is transitive.

B R_2 is transitive but R_1 is not transitive.

C R_1 and R_2 are both transitive.

D R_1 is transitive but R_2 is not transitive.

Explanation

For R_1 :

Let $a = 1 + \sqrt{2}$, $b = 1 - \sqrt{2}$, $c = 8\frac{1}{4}$

$aR_1b : a^2 + b^2 = 6 \in Q$

$bR_1c : b^2 + c^2 = 3 - 2\sqrt{2} + 2\sqrt{2} = 3 \in Q$

$aR_1c : a^2 + c^2 = 3 + 2\sqrt{2} + 2\sqrt{2} \notin Q$

$\therefore R_1$ is not transitive.

For R_2 :

Let $a = 1 + \sqrt{2}$, $b = \sqrt{2}$, $c = 1 - \sqrt{2}$

$aR_2b : a^2 + b^2 = 5 + 2\sqrt{2} \notin Q$

$bR_2c : b^2 + c^2 = 5 - 2\sqrt{2} \notin Q$

$aR_2c : a^2 + c^2 = 3 + 2\sqrt{2} + 3 - 2\sqrt{2} = 6 \in Q$

$\therefore R_2$ is not transitive.

Again diferent types of relations definition is used to solve this question.

1 JEE Main 2020 (Online) 2nd September Morning Slot

MCQ (Single Correct Answer)

If $R = \{(x, y) : x, y \in Z, x^2 + 3y^2 \leq 8\}$ is a relation on the set of integers Z, then the domain of R^{-1} is :

A $\{0, 1\}$

B $\{-2, -1, 1, 2\}$

C $\{-1, 0, 1\}$

D $\{-2, -1, 0, 1, 2\}$

Explanation

Given $R = \{(x, y) : x, y \in Z, x^2 + 3y^2 \leq 8\}$

So R = {(0,1), (0,-1), (1,0), (-1,0), (1,1), (1,-1)
(-1,1), (-1,-1), (2,0), (-2,0), (-2,0), (2,1), (2,-1), (-2,1), (-2,-1)}

$\Rightarrow$ R : { -2, -1, 0, 1, 2} $\rightarrow$ {-1, 0, 1}

$\therefore R^{-1}$: {-1, 0, 1} $\rightarrow$ { -2, -1, 0, 1, 2}

$\therefore$ Domain of R^{-1} = {-1, 0, 1}

4 JEE Main 2018 (Online) 16th April Morning Slot

MCQ (Single Correct Answer)

Let N denote the set of all natural numbers. Define two binary relations on N as $R_1 = \{(x, y) \in N \times N : 2x + y = 10\}$ and $R_2 = \{(x, y) \in N \times N : x + 2y = 10\}$. Then :

A Range of R_1 is {2, 4, 8).

B Range of R_2 is {1, 2, 3, 4}.

C Both R_1 and R_2 are symmetric relations.

D Both R_1 and R_2 are transitive relations.

Explanation

For R_1; $2x + y = 10$ and $x, y \in N$ possible values for x and y are :

$x = 1, y = 8$ i.e. (1, 8);

$x = 2, y = 6$ i.e (2, 6);

$x = 3, y = 4$ i.e (3, 4);

$x = 4, y = 2$ i.e (4, 2)

$\therefore R_1 = \{ (1, 8), (2, 6), (3, 4), (4, 2) \}$

$\therefore$ Range of R_1 is {2, 4, 6, 8}

R_1 is not symmetric.

R_1 is not transitive also as

$(3, 4), (4, 2) \in R$, but $(3, 2) \notin R_1$

For R_2 : $x + 2y = 10$ and $x, y \in N$

Possible values of x, and y are :

$x = 8, y = 1$ i.e (8, 1)

$x = 6, y = 2$ i.e (6, 2)

$x = 4, y = 3$ i.e (4, 3) and

$x = 2, y = 4$ i.e (2, 4)

$\therefore R_2 = \{(8, 1) (6, 2) (4, 3) (2, 4)\}$

$\therefore$ Range of $R_2 = \{1,2,3,4\}$

R_2 is not symmetric and transitive