3.5 Drift of Electrons and the Origin of Resistivity

As remarked before, an electron will suffer collisions with the heavy fixed ions, but after collision, it will emerge with the same speed but in random directions. If we consider all the electrons, their average velocity will be zero since their directions are random. Thus, if there are N electrons and the velocity of the i^{th} electron (i = 1, 2, 3, ... N) at a given time is $\mathbf{v}_i$, then

$$\frac{1}{N}\sum_{i=1}^{N}\mathbf{v}_i = 0 \tag{3.14}$$

Consider now the situation when an electric field is present. Electrons will be accelerated due to this field by

$$\mathbf{a} = \frac{-e\,\mathbf{E}}{m} \tag{3.15}$$

where $-e$ is the charge and m is the mass of an electron. Consider again the i^{th} electron at a given time t. This electron would have had its last collision some time before t, and let t_i be the time elapsed after its last collision. If $\mathbf{v}_i$ was its velocity immediately after the last collision, then its velocity $\mathbf{V}_i$ at time t is

$$\mathbf{V}_i = \mathbf{v}_i + \frac{-e\mathbf{E}}{m}\,t_i \tag{3.16}$$

since starting with its last collision it was accelerated (Fig. 3.3) with an acceleration given by Eq. (3.15) for a time interval t_i. The average velocity of the electrons at time t is the average of all the $\mathbf{V}_i$'s. The average of $\mathbf{v}_i$'s is zero [Eq. (3.14)] since immediately after any collision, the direction of the velocity of an electron is completely random. The collisions of the electrons do not occur at regular intervals but at random times. Let us denote by τ, the average time between successive collisions. Then at a given time, some of the electrons would have spent

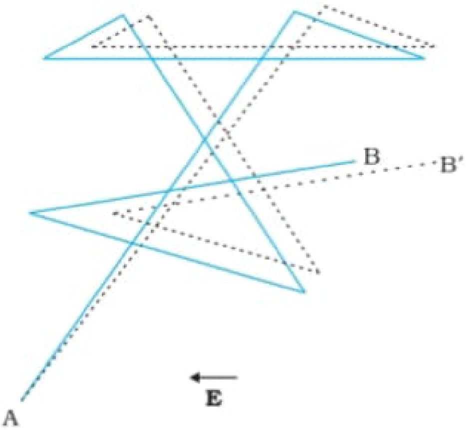

FIGURE 3.3 A schematic picture of an electron moving from a point A to another point B through repeated collisions, and straight line travel between collisions (full lines). If an electric field is applied as shown, the electron ends up at point B′ (dotted lines). A slight drift in a direction opposite the electric field is visible.

time more than τ and some less than τ. In other words, the time t_i in Eq. (3.16) will be less than τ for some and more than τ for others as we go through the values of i = 1, 2 ..... N. The average value of t_i then is τ (known as *relaxation time*). Thus, averaging Eq. (3.16) over the N-electrons at any given time **t** gives us for the average velocity $\mathbf{v}_d$

$$\mathbf{v}_d \equiv (\mathbf{V}_i)_{average} = (\mathbf{v}_i)_{average} - \frac{e\mathbf{E}}{m}(t_i)_{average}$$

$$= 0 - \frac{e\mathbf{E}}{m}\tau = -\frac{e\mathbf{E}}{m}\tau \tag{3.17}$$

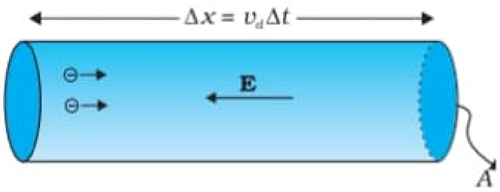

FIGURE 3.4 Current in a metallic conductor. The magnitude of current density in a metal is the magnitude of charge contained in a cylinder of unit area and length v_d.

This last result is surprising. It tells us that the electrons move with an average velocity which is independent of time, although electrons are accelerated. This is the phenomenon of drift and the velocity $\mathbf{v}_d$ in Eq. (3.17) is called the ***drift velocity***.

Because of the drift, there will be net transport of charges across any area perpendicular to **E**. Consider a planar area *A*, located inside the conductor such that the normal to the area is parallel to **E** (Fig. 3.4). Then because of the drift, in an infinitesimal amount of time Δ***t***, all electrons to the left of the area at distances upto $|\mathbf{v}_d|\Delta t$ would have crossed the area. If ***n*** is the number of free electrons per unit volume in the metal, then there are $n\ \Delta t\ |\mathbf{v}_d| A$ such electrons. Since each electron carries a charge –***e***, the total charge transported across this area **A** to the right in time Δ*t* is $-neA|\mathrm{v}_d|\Delta t$. **E** is directed towards the left and hence the total charge transported along **E** across the area is negative of this. The amount of charge crossing the area *A* in time Δ*t* is by definition [Eq. (3.2)] *I* Δ*t*, where *I* is the magnitude of the current. Hence,

$$I\,\Delta t = +n\,e\,A\,|\mathrm{v}_d|\Delta t \tag{3.18}$$

Substituting the value of $|\mathbf{v}_d|$ from Eq. (3.17)

$$I\Delta t = \frac{e^2 A}{m}\tau n\,\Delta t|\mathrm{E}| \tag{3.19}$$

By definition ***I*** is related to the magnitude |j| of the current density by

$$I = |\mathrm{j}|A \tag{3.20}$$

Hence, from Eqs.(3.19) and (3.20),

$$|\mathrm{j}| = \frac{ne^2}{m}\tau|\mathrm{E}| \tag{3.21}$$

The vector **j** is parallel to **E** and hence we can write Eq. (3.21) in the vector form

$$\mathbf{j} = \frac{ne^2}{m}\tau\mathbf{E} \tag{3.22}$$

Comparison with Eq. (3.13) shows that Eq. (3.22) is exactly the Ohm's law, if we identify the conductivity σ as

$$\sigma = \frac{ne^2}{m}\tau \qquad (3.23)$$

We thus see that a very simple picture of electrical conduction reproduces Ohm's law. We have, of course, made assumptions that τ and n are constants, independent of E. We shall, in the next section, discuss the limitations of Ohm's law.

Example 3.1 (a) Estimate the average drift speed of conduction electrons in a copper wire of cross-sectional area 1.0×10^{-7} m^2 carrying a current of 1.5 A. Assume that each copper atom contributes roughly one conduction electron. The density of copper is 9.0×10^3 kg/m^3, and its atomic mass is 63.5 u. (b) Compare the drift speed obtained above with, (i) thermal speeds of copper atoms at ordinary temperatures, (ii) speed of propagation of electric field along the conductor which causes the drift motion.

Solution

(a) The direction of drift velocity of conduction electrons is opposite to the electric field direction, i.e., electrons drift in the direction of increasing potential. The drift speed v_d is given by Eq. (3.18)
$v_d = (I/neA)$
Now, $e = 1.6 \times 10^{-19}$ C, $A = 1.0 \times 10^{-7}\text{m}^2$, $I = 1.5$ A. The density of conduction electrons, n is equal to the number of atoms per cubic metre (assuming one conduction electron per Cu atom as is reasonable from its valence electron count of one). A cubic metre of copper has a mass of 9.0×10^3 kg. Since 6.0×10^{23} copper atoms have a mass of 63.5 g,

$$n = \frac{6.0 \times 10^{23}}{63.5} \times 9.0 \times 10^6$$

$$= 8.5 \times 10^{28}\ \text{m}^{-3}$$

which gives,

$$v_d = \frac{1.5}{8.5 \times 10^{28} \times 1.6 \times 10^{-19} \times 1.0 \times 10^{-7}}$$

$$= 1.1 \times 10^{-3}\ \text{m s}^{-1} = 1.1\ \text{mm s}^{-1}$$

(b) (i) At a temperature T, the thermal speed* of a copper atom of mass M is obtained from [$<(1/2)\, Mv^2> = (3/2)\, k_B T$] and is thus typically of the order of $\sqrt{k_B T/M}$, where k_B is the Boltzmann constant. For copper at 300 K, this is about 2×10^2 m/s. This figure indicates the random vibrational speeds of copper atoms in a conductor. Note that the drift speed of electrons is much smaller, about 10^{-5} times the typical thermal speed at ordinary temperatures.
(ii) An electric field travelling along the conductor has a speed of an electromagnetic wave, namely equal to 3.0×10^8 m s^{-1} (You will learn about this in Chapter 8). The drift speed is, in comparison, extremely small; smaller by a factor of 10^{-11}.

EXAMPLE 3.1

* See Eq. (13.23) of Chapter 13 from Class XI book.

3.4 Ohm's Law

A basic law regarding flow of currents was discovered by G.S. Ohm in 1828, long before the physical mechanism responsible for flow of currents was discovered. Imagine a conductor through which a current I is flowing and let V be the potential difference between the ends of the conductor. Then Ohm's law states that

$$V \propto I$$

$$\text{or, } V = R\,I \tag{3.3}$$

where the constant of proportionality R is called the *resistance* of the conductor. The SI units of resistance is *ohm*, and is denoted by the symbol Ω. The resistance R not only depends on the material of the conductor but also on the dimensions of the conductor. The dependence of R on the dimensions of the conductor can easily be determined as follows.

Consider a conductor satisfying Eq. (3.3) to be in the form of a slab of length l and cross sectional area A [Fig. 3.2(a)]. Imagine placing two such identical slabs side by side [Fig. 3.2(b)], so that the length of the combination is $2l$. The current flowing through the combination is the same as that flowing through either of the slabs. If V is the potential difference across the ends of the first slab, then V is also the potential difference across the ends of the second slab since the second slab is

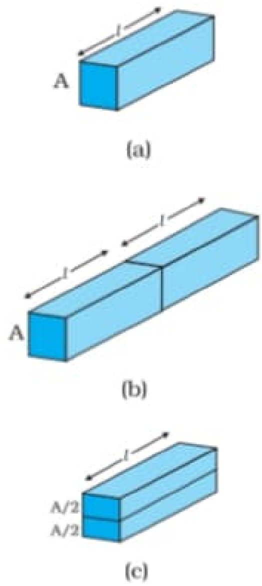

FIGURE 3.2 Illustrating the relation $R = \rho l/A$ for a rectangular slab of length l and area of cross-section A.

GEORG SIMON OHM (1787–1854)

Georg Simon Ohm (1787–1854) German physicist, professor at Munich. Ohm was led to his law by an analogy between the conduction of heat: the electric field is analogous to the temperature gradient, and the electric current is analogous to the heat flow.

identical to the first and the same current I flows through both. The potential difference across the ends of the combination is clearly sum of the potential difference across the two individual slabs and hence equals $2V$. The current through the combination is I and the resistance of the combination R_C is [from Eq. (3.3)],

$$R_C = \frac{2V}{I} = 2R \tag{3.4}$$

since $V/I = R$, the resistance of either of the slabs. Thus, doubling the length of a conductor doubles the resistance. In general, then resistance is proportional to length,

$$R \propto l \tag{3.5}$$

Next, imagine dividing the slab into two by cutting it lengthwise so that the slab can be considered as a combination of two identical slabs of length l, but each having a cross sectional area of $A/2$ [Fig. 3.2(c)].

For a given voltage V across the slab, if I is the current through the entire slab, then clearly the current flowing through each of the two half-slabs is $I/2$. Since the potential difference across the ends of the half-slabs is V, i.e., the same as across the full slab, the resistance of each of the half-slabs R_1 is

$$R_1 = \frac{V}{(I/2)} = 2\frac{V}{I} = 2R. \tag{3.6}$$

Thus, halving the area of the cross-section of a conductor doubles the resistance. In general, then the resistance R is inversely proportional to the cross-sectional area,

$$R \propto \frac{1}{A} \tag{3.7}$$

Combining Eqs. (3.5) and (3.7), we have

$$R \propto \frac{l}{A} \tag{3.8}$$

and hence for a given conductor

$$R = \rho\,\frac{l}{A} \tag{3.9}$$

where the constant of proportionality ρ depends on the material of the conductor but not on its dimensions. ρ is called *resistivity*.

Using the last equation, Ohm's law reads

$$V = I \times R = \frac{I\rho l}{A} \tag{3.10}$$

Current per unit area (taken normal to the current), I/A, is called *current density* and is denoted by j. The SI units of the current density are A/m^2. Further, if E is the magnitude of uniform electric field in the conductor whose length is l, then the potential difference V across its ends is El. Using these, the last equation reads

Example 3.2

(a) In Example 3.1, the electron drift speed is estimated to be only a few mm s^{-1} for currents in the range of a few amperes? How then is current established almost the instant a circuit is closed?

(b) The electron drift arises due to the force experienced by electrons in the electric field inside the conductor. But force should cause acceleration. Why then do the electrons acquire a steady average drift speed?

(c) If the electron drift speed is so small, and the electron's charge is small, how can we still obtain large amounts of current in a conductor?

(d) When electrons drift in a metal from lower to higher potential, does it mean that all the 'free' electrons of the metal are moving in the same direction?

(e) Are the paths of electrons straight lines between successive collisions (with the positive ions of the metal) in the (i) absence of electric field, (ii) presence of electric field?

Solution

(a) Electric field is established throughout the circuit, almost instantly (with the speed of light) causing at every point a *local electron drift*. Establishment of a current does not have to wait for electrons from one end of the conductor travelling to the other end. However, it does take a little while for the current to reach its steady value.

(b) Each 'free' electron does accelerate, increasing its drift speed until it collides with a positive ion of the metal. It loses its drift speed after collision but starts to accelerate and increases its drift speed again only to suffer a collision again and so on. On the average, therefore, electrons acquire only a drift speed.

(c) Simple, because the electron number density is enormous, $\sim 10^{29}$ m^{-3}.

(d) By no means. The drift velocity is superposed over the large random velocities of electrons.

(e) In the absence of electric field, the paths are straight lines; in the presence of electric field, the paths are, in general, curved.

EXAMPLE 3.2

3.7 Resistivity of Various Materials

The resistivities of various common materials are listed in Table 3.1. The materials are classified as conductors, semiconductors and insulators depending on their resistivities, in an increasing order of their values. Metals have low resistivities in the range of 10^{-8} Ωm to 10^{-6} Ωm. At the other end are insulators like ceramic, rubber and plastics having resistivities 10^{18} times greater than metals or more. In between the two are the semiconductors. These, however, have resistivities characteristically decreasing with a rise in temperature. The resistivities of semiconductors can be decreased by adding small amount of suitable impurities. This last feature is exploited in use of semiconductors for electronic devices.

TABLE 3.1 RESISTIVITIES OF SOME MATERIALS

Material	**Resistivity,** ρ (Ω m) **at** 0°C	**Temperature coefficient of resistivity,** α (°C)$^{-1}$ $\frac{1}{\rho}\frac{d\rho}{dT}$ at 0°C
Conductors		
Silver	1.6×10^{-8}	0.0041
Copper	1.7×10^{-8}	0.0068
Aluminium	2.7×10^{-8}	0.0043
Tungsten	5.6×10^{-8}	0.0045
Iron	10×10^{-8}	0.0065
Platinum	11×10^{-8}	0.0039
Mercury	98×10^{-8}	0.0009
Nichrome (alloy of Ni, Fe, Cr)	$\sim 100 \times 10^{-8}$	0.0004
Manganin (alloy)	48×10^{-8}	0.002×10^{-3}
Semiconductors		
Carbon (graphite)	3.5×10^{-5}	– 0.0005
Germanium	0.46	– 0.05
Silicon	2300	– 0.07
Insulators		
Pure Water	2.5×10^{5}	
Glass	$10^{10} - 10^{14}$	
Hard Rubber	$10^{13} - 10^{16}$	
NaCl	$\sim 10^{14}$	
Fused Quartz	$\sim 10^{16}$	

Commercially produced resistors for domestic use or in laboratories are of two major types: *wire bound resistors* and *carbon resistors*. Wire bound resistors are made by winding the wires of an alloy, viz., manganin, constantan, nichrome or similar ones. The choice of these materials is dictated mostly by the fact that their resistivities are relatively insensitive to temperature. These resistances are typically in the range of a fraction of an ohm to a few hundred ohms.