Alternatively, we can proceed as follows:

$$y = \sin(\cos x^2)$$

Therefore $$\frac{dy}{dx} = \frac{d}{dx}\sin(\cos x^2) = \cos(\cos x^2)\frac{d}{dx}(\cos x^2)$$

$$= \cos(\cos x^2)(-\sin x^2)\frac{d}{dx}(x^2)$$

$$= -\sin x^2 \cos(\cos x^2)(2x)$$
$$= -2x\sin x^2 \cos(\cos x^2)$$

EXERCISE 5.2

Differentiate the functions with respect to x in Exercises 1 to 8.

1. $\sin(x^2 + 5)$
2. $\cos(\sin x)$
3. $\sin(ax + b)$
4. $\sec(\tan(\sqrt{x}))$
5. $\dfrac{\sin(ax+b)}{\cos(cx+d)}$
6. $\cos x^3 . \sin^2(x^5)$
7. $2\sqrt{\cot(x^2)}$
8. $\cos(\sqrt{x})$
9. Prove that the function f given by
$$f(x) = |x - 1|,\ x \in \mathbf{R}$$
is not differentiable at $x = 1$.
10. Prove that the greatest integer function defined by
$$f(x) = [x],\ 0 < x < 3$$
is not differentiable at $x = 1$ and $x = 2$.

5.3.2 *Derivatives of implicit functions*

Until now we have been differentiating various functions given in the form $y = f(x)$. But it is not necessary that functions are always expressed in this form. For example, consider one of the following relationships between x and y:

$$x - y - \pi = 0$$

$$x + \sin xy - y = 0$$

In the first case, we can *solve for* y and rewrite the relationship as $y = x - \pi$. In the second case, it does not seem that there is an easy way to *solve for* y. Nevertheless, there is no doubt about the dependence of y on x in either of the cases. When a relationship between x and y is expressed in a way that it is easy to *solve for* y and write $y = f(x)$, we say that y is given as an *explicit function* of x. In the latter case it is implicit that y is a function of x and we say that the relationship of the second type, above, gives function *implicitly*. In this subsection, we learn to differentiate implicit functions.

Example 24 Find $\frac{dy}{dx}$ if $x - y = \pi$.

Solution One way is to solve for y and rewrite the above as

$$y = x - \pi$$

But then $$\frac{dy}{dx} = 1$$

Alternatively, *directly* differentiating the relationship w.r.t., x, we have

$$\frac{d}{dx}(x - y) = \frac{d\pi}{dx}$$

Recall that $\frac{d\pi}{dx}$ means to differentiate the constant function taking value π everywhere w.r.t., x. Thus

$$\frac{d}{dx}(x) - \frac{d}{dx}(y) = 0$$

which implies that

$$\frac{dy}{dx} = \frac{dx}{dx} = 1$$

Example 25 Find $\frac{dy}{dx}$, if $y + \sin y = \cos x$.

Solution We differentiate the relationship directly with respect to x, i.e.,

$$\frac{dy}{dx} + \frac{d}{dx}(\sin y) = \frac{d}{dx}(\cos x)$$

which implies using chain rule

$$\frac{dy}{dx} + \cos y \cdot \frac{dy}{dx} = -\sin x$$

This gives $$\frac{dy}{dx} = -\frac{\sin x}{1 + \cos y}$$

where $$y \neq (2n + 1)\pi$$