Question -

If C_r stands for ${}^nC_r$, then the sum of the series

$$\frac{2\left(\frac{n}{2}\right)!\left(\frac{n}{2}\right)!}{n!}[C_0^2 - 2C_1^2 + 3C_2^2 - \ldots + (-1)^n(n+1)C_n^2],$$

where n is an even positive integer, is **(1986, 2M)**

(a) $(-1)^{n/2}(n+2)$ (b) $(-1)^n(n+1)$

(c) $(-1)^{n/2}(n+1)$ (d) None of these

Ans - A

Solution -

We have,

$$C_0^2 - 2C_1^2 + 3C_2^2 - 4C_3^2 + \ldots + (-1)^n(n+1)C_n^2$$

$$= [C_0^2 - C_1^2 + C_2^2 - C_3^2 + \ldots + (-1)^n C_n^2] - [C_1^2 - 2C_2^2 + 3C_3^2 - \ldots + (-1)^n nC_n^2]$$

$$= (-1)^{n/2}\frac{n!}{\left(\frac{n}{2}\right)!\left(\frac{n}{2}\right)!} - (-1)^{\frac{n}{2}-1}\frac{n}{2}\frac{n!}{\left(\frac{n}{2}\right)!\left(\frac{n}{2}\right)!}$$

$$= (-1)^{n/2}\frac{n!}{\left(\frac{n}{2}\right)!\left(\frac{n}{2}\right)!}\left(1+\frac{n}{2}\right)$$

$$\therefore \frac{2\left(\frac{n}{2}\right)!\left(\frac{n}{2}\right)!}{n!}[C_0^2 - 2C_1^2 + 3C_2^2 - \ldots + (-1)^r(n+1)C_n^2]$$

$$= \frac{2\left(\frac{n}{2}\right)!\left(\frac{n}{2}\right)!}{n!}(-1)^{n/2}\frac{n!}{\left(\frac{n}{2}\right)!\left(\frac{n}{2}\right)!}\frac{(n+2)}{2} = (-1)^{n/2}(n+2)$$