

Question -

If $a_n = \sum_{r=0}^n \frac{1}{{}^n C_r}$, then $\sum_{r=0}^n \frac{r}{{}^n C_r}$ equals

- (a) $(n - 1) a_n$ (b) $n a_n$ (1998, 2M)
(c) $\frac{1}{2} n a_n$ (d) None of these

Ans - C

Solution -

$$\begin{aligned} \text{Let } b &= \sum_{r=0}^n \frac{r}{{}^n C_r} = \sum_{r=0}^n \frac{n - (n - r)}{{}^n C_r} \\ &= n \sum_{r=0}^n \frac{1}{{}^n C_r} - \sum_{r=0}^n \frac{n - r}{{}^n C_r} \\ &= n a_n - \sum_{r=0}^n \frac{n - r}{{}^n C_{n-r}} \quad [\because {}^n C_r = {}^n C_{n-r}] \\ &= n a_n - b \Rightarrow 2b = n a_n \Rightarrow b = \frac{n}{2} a_n \end{aligned}$$