Question -

The sum $\sum\limits_{i=0}^{m} \dbinom{10}{i}\dbinom{20}{m-i}$, where $\dbinom{p}{q}=0$ if $p>q$, is maximum when m is equal to   (2002, 1M)

(a) 5   (b) 10   (c) 15   (d) 20

Ans - C

Solution -

$\sum\limits_{i=0}^{m} \dbinom{10}{i}\dbinom{20}{m-i}$ is the coefficient of x^m in the expansion of $(1 + x)^{10}(x + 1)^{20}$,

$\Rightarrow \sum\limits_{i=0}^{m} \dbinom{10}{i}\dbinom{20}{m-i}$ is the coefficient of x^m in the

expansion of $(1 + x)^{30}$

i.e. $\sum\limits_{i=0}^{m} \dbinom{10}{i}\dbinom{20}{m-i} = {}^{30}C_m = \dbinom{30}{m}$ ...(i)

and we know that, $\dbinom{n}{r}$ is maximum, when

$$\dbinom{n}{r}_{max} = \begin{cases} r = \dfrac{n}{2}, & \text{if } n \in \text{even.} \\ r = \dfrac{n \pm 1}{2}, & \text{if } n \in \text{odd.} \end{cases}$$

Hence, $\dbinom{30}{m}$ is maximum when $m = 15$.