Question -

$\binom{30}{0}\binom{30}{10}-\binom{30}{1}\binom{30}{11}+\binom{30}{2}\binom{30}{12}+\ldots+\binom{30}{20}\binom{30}{30}$ is equal to **(2005, 1M)**

(a) ${}^{30}C_{11}$ (b) ${}^{60}C_{10}$

(c) ${}^{30}C_{10}$ (d) ${}^{65}C_{55}$

Ans - C

Solution -

$$\text{Let } A=\binom{30}{0}\binom{30}{10}-\binom{30}{1}\binom{30}{11}+\binom{30}{2}\binom{30}{12}-\ldots+\binom{30}{20}\binom{30}{30}$$

$$\therefore \quad A = {}^{30}C_0 \cdot {}^{30}C_{10} - {}^{30}C_1 \cdot {}^{30}C_{11} + {}^{30}C_2 \cdot {}^{30}C_{12} - \ldots + {}^{30}C_{20} \cdot {}^{30}C_{30}$$

$$= \text{Coefficient of } x^{20} \text{ in } (1+x)^{30}(1-x)^{30}$$

$$= \text{Coefficient of } x^{20} \text{ in } (1-x^2)^{30}$$

$$= \text{Coefficient of } x^{20} \text{ in } \sum_{r=0}^{30} (-1)^r {}^{30}C_r (x^2)^r$$

$$= (-1)^{10}\ {}^{30}C_{10} \qquad [\text{for coefficient of } x^{20}, \text{ put } r = 10]$$

$$= {}^{30}C_{10}$$