Question -

The value of r for which

$${}^{20}C_r\ {}^{20}C_0 + {}^{20}C_{r-1}\ {}^{20}C_1 + {}^{20}C_{r-2}\ {}^{20}C_2 + \ldots + {}^{20}C_0\ {}^{20}C_r$$

is maximum, is **(2019 Main, 11 Jan I)**

(a) 15 (b) 10 (c) 11 (d) 20

Ans - D

Solution -

We know that,

$$(1 + x)^{20} = {}^{20}C_0 + {}^{20}C_1 x + {}^{20}C_2 x^2 + \ldots + {}^{20}C_{r-1} x^{r-1} + {}^{20}C_r x^r + \ldots + {}^{20}C_{20} x^{20}$$

$$\therefore\ (1 + x)^{20} \cdot (1 + x)^{20} = ({}^{20}C_0 + {}^{20}C_1 x + {}^{20}C_2 x^2 + \ldots + {}^{20}C_{r-1} x^{r-1} + {}^{20}C_r x^r + \ldots + {}^{20}c_{20} x^{20})$$

$$\times ({}^{20}C_0 + {}^{20}C_1 x + \ldots + {}^{20}C_{r-1} x^{r-1} + {}^{20}C_r x^r + \ldots + {}^{20}C_{20} x^{20})$$

$$\Rightarrow (1 + x)^{40} = ({}^{20}C_0\ .\ {}^{20}C_r + {}^{20}C_1\ {}^{20}C_{r-1} \ldots {}^{20}C_r\ {}^{20}C_0)\ x^r + \ldots$$

On comparing the coefficient of x^r of both sides, we get

$${}^{20}C_0\ {}^{20}C_r + {}^{20}C_1\ {}^{20}C_{r-1} + \ldots + {}^{20}C_r\ {}^{20}C_0 = {}^{40}C_r$$

The maximum value of ${}^{40}C_r$ is possible only when $r = 20$

[$\because {}^nC_{n/2}$ is maximum when n is even]

Thus, required value of r is 20.