Question -

If the coefficients of x^3 and x^4 in the expansion of $(1 + ax + bx^2)(1 - 2x)^{18}$ in powers of x are both zero, then (a, b) is equal to

(a) $\left(16, \frac{251}{3}\right)$ (b) $\left(14, \frac{251}{3}\right)$

(c) $\left(14, \frac{272}{3}\right)$ (d) $\left(16, \frac{272}{3}\right)$

Ans - D

Solution -

To find the coefficient of x^3 and x^4, use the formula of coefficient of x^r in $(1 - x)^n$ is $(-1)^r \, {}^nC_r$ and then simplify.

In expansion of $(1 + ax + bx^2)(1 - 2x)^{18}$.

$$\text{Coefficient of } x^3 = \text{Coefficient of } x^3 \text{ in } (1 - 2x)^{18}$$
$$+ \text{Coefficient of } x^2 \text{ in } a\,(1 - 2x)^{18}$$
$$+ \text{Coefficient of } x \text{ in } b(1 - 2x)^{18}$$
$$= {}^{18}C_3 \cdot 2^3 + a\,{}^{18}C_2 \cdot 2^2 - b\,{}^{18}C_1 \cdot 2$$

Given, coefficient of $x^3 = 0$

$$\Rightarrow \quad {}^{18}C_3 \cdot 2^3 + a\,{}^{18}C_2 \cdot 2^2 - b\,{}^{18}C_1 \cdot 2 = 0$$

$$\Rightarrow \quad -\frac{18 \times 17 \times 16}{3 \times 2} \cdot 8 + a \cdot \frac{18 \times 17}{2} \cdot 2^2 - b \cdot 18 \cdot 2 = 0$$

$$\Rightarrow \quad 17a - b = \frac{34 \times 16}{3} \qquad ..(i)$$

Similarly, coefficient of $x^4 = 0$

$$\Rightarrow \quad {}^{18}C_4 \cdot 2^4 - a \cdot {}^{18}C_3 2^3 + b \cdot {}^{18}C_2 \cdot 2^2 = 0$$

$$\therefore \quad 32a - 3b = 240 \qquad \ldots(ii)$$

On solving Eqs. (i) and (ii), we get

$$a = 16, \; b = \frac{272}{3}$$