

Question -

The term independent of x in expansion of

$$\left(\frac{x+1}{x^{2/3} - x^{1/3} + 1} - \frac{x-1}{x - x^{1/2}} \right)^{10} \text{ is} \quad \text{(2013 Main)}$$

- (a) 4 (b) 120 (c) 210 (d) 310

Ans - C

Solution -

$$\begin{aligned} & \left[\frac{x+1}{x^{2/3} - x^{1/3} + 1} - \frac{(x-1)}{x - x^{1/2}} \right]^{10} \\ &= \left[\frac{(x^{1/3})^3 + 1^3}{x^{2/3} - x^{1/3} + 1} - \frac{\{(\sqrt{x})^2 - 1\}}{\sqrt{x}(\sqrt{x} - 1)} \right]^{10} \\ &= \left[\frac{(x^{1/3} + 1)(x^{2/3} + 1 - x^{1/3})}{x^{2/3} - x^{1/3} + 1} - \frac{\{(\sqrt{x})^2 - 1\}}{\sqrt{x}(\sqrt{x} - 1)} \right]^{10} \\ &= \left[(x^{1/3} + 1) - \frac{(\sqrt{x} + 1)}{\sqrt{x}} \right]^{10} = (x^{1/3} - x^{-1/2})^{10} \end{aligned}$$

∴ The general term is

$$T_{r+1} = {}^{10}C_r (x^{1/3})^{10-r} (-x^{-1/2})^r = {}^{10}C_r (-1)^r x^{\frac{10-r}{3} - \frac{r}{2}}$$

For independent of x , put

$$\frac{10-r}{3} - \frac{r}{2} = 0 \quad \Rightarrow \quad 20 - 2r - 3r = 0$$

$$\Rightarrow \quad 20 = 5r \quad \Rightarrow \quad r = 4$$

$$\therefore T_5 = {}^{10}C_4 = \frac{10 \times 9 \times 8 \times 7}{4 \times 3 \times 2 \times 1} = 210$$