Question -

The term independent of x in the expansion of $\left(\frac{1}{60}-\frac{x^8}{81}\right)\cdot\left(2x^2-\frac{3}{x^2}\right)^6$ is equal to **(2019 Main, 12 April II)**

(a) – 72 (b) 36 (c) – 36 (d) – 108

Ans - C

Solution -

Let a binomial $\left(2x^2-\frac{3}{x^2}\right)^6$, it's $(r+1)$th term

$$= T_{r+1} = {}^6C_r\,(2x^2)^{6-r}\left(-\frac{3}{x^2}\right)^r$$

$$= {}^6C_r\,(-3)^r\,(2)^{6-r}\,x^{12-2r-2r}$$

$$= {}^6C_r\,(-3)^r\,(2)^{6-r}\,x^{12-4r} \quad \ldots\text{(i)}$$

Now, the term independent of x in the expansion of $\left(\frac{1}{60}-\frac{x^8}{81}\right)\left(2x^2-\frac{3}{x^2}\right)^6$

= the term independent of x in the expansion of $\frac{1}{60}\left(2x^2-\frac{3}{x^2}\right)^6$ + the term independent of x in the expansion of $-\frac{x^8}{81}\left(2x^2-\frac{3}{x^2}\right)^6$

$$= \frac{{}^6C_3}{60}(-3)^3\,(2)^{6-3}\,x^{12-4(3)} \quad [\text{put } r=3]$$

$$+\left(-\frac{1}{81}\right){}^6C_5\,(-3)^5\,(2)^{6-5}\,x^{12-4(5)}\,x^8 \quad [\text{put } r=5]$$

$$= \frac{1}{3}(-3)^3\,2^3 + \frac{3^5\times 2(6)}{81}$$

$$= 36 - 72 = -36$$