Example 1 Let A = {1, 2, 3, 4} and B = {5, 7, 9}. Determine

(i) $A \times B$ (ii) $B \times A$

(iii) Is $A \times B = B \times A$? (iv) Is $n(A \times B) = n(B \times A)$?

Solution Since A = {1, 2, 3, 4} and B = {5, 7, 9}. Therefore,

(i) $A \times B = \{(1, 5), (1, 7), (1, 9), (2, 5), (2, 7),$
$(2, 9), (3, 5), (3, 7), (3, 9), (4, 5), (4, 7), (4, 9)\}$

(ii) $B \times A = \{(5, 1), (5, 2), (5, 3), (5, 4), (7, 1), (7, 2),$
$(7, 3), (7, 4), (9, 1), (9, 2), (9, 3), (9, 4)\}$

(iii) No, $A \times B \neq B \times A$. Since $A \times B$ and $B \times A$ do not have exactly the same ordered pairs.

(iv) $n(A \times B) = n(A) \times n(B) = 4 \times 3 = 12$

$n(B \times A) = n(B) \times n(A) = 4 \times 3 = 12$

Hence $n(A \times B) = n(B \times A)$

Example 2 Find x and y if:

(i) $(4x + 3, y) = (3x + 5, -2)$ (ii) $(x - y, x + y) = (6, 10)$

Solution

(i) Since $(4x + 3, y) = (3x + 5, -2)$, so

$4x + 3 = 3x + 5$

or $x = 2$

and $y = -2$

(ii) $x - y = 6$

$x + y = 10$

$\therefore$ $2x = 16$

or $x = 8$

$8 - y = 6$

$\therefore$ $y = 2$

Example 3 If A = {2, 4, 6, 9} and B = {4, 6, 18, 27, 54}, $a \in$ A, $b \in$ B, find the set of ordered pairs such that 'a' is factor of 'b' and $a < b$.

Solution Since A = {2, 4, 6, 9}

B = {4, 6, 18, 27, 54},

we have to find a set of ordered pairs (a, b) such that a is factor of b and $a < b$.

Since 2 is a factor of 4 and 2 < 4.

So (2, 4) is one such ordered pair.

Similarly, (2, 6), (2, 18), (2, 54) are other such ordered pairs. Thus the required set of ordered pairs is

{(2, 4), (2, 6), (2, 18), (2, 54), (6, 18), (6, 54,), (9, 18), (9, 27), (9, 54)}.

1. Let A = {-1, 2, 3} and B = {1, 3}. Determine
(i) A × B
(ii) B × A
(iii) B × B
(iv) A × A

Solution:

According to the question,

A = {-1, 2, 3} and B = {1, 3}

(i) A × B

{-1, 2, 3} × {1, 3}

So, A × B = {(-1, 1), (-1, 3), (2, 1), (2, 3), (3, 1), (3, 3)}

Hence, the Cartesian product = {(-1, 1), (-1, 3), (2, 1), (2, 3), (3, 1), (3, 3)}

(ii) B × A.

{1, 3} × {-1, 2, 3}

So, B × A = {(1, -1), (1, 2), (1, 3), (3, -1), (3, 2), (3, 3)}

Hence, the Cartesian product = {(1, -1), (1, 2), (1, 3), (3, -1), (3, 2), (3, 3)}

(iii) B × B

{1, 3} ×{1, 3}

So, B × B = {(1, 1), (1, 3), (3, 1), (3, 3)}

Hence, the Cartesian product = {(1, 1), (1, 3), (3, 1), (3, 3)}

(iv) A × A

{-1, 2, 3} × {-1, 2, 3}

So, A × A = {(-1, -1), (-1, 2), (-1, 3), (2, -1), (2, 2), (2, 3), (3, -1), (3, 2), (3, 3)}

Hence,

the Cartesian product ={(-1, -1), (-1, 2), (-1, 3), (2, -1), (2, 2), (2, 3), (3, -1), (3, 2), (3, 3)}

39.If

$$P = \{1, 2\},$$

then

$$P \times P \times P = \{(1,1,1), (2,2,2), (1,2,2), (2,1,1)\}.$$

Ans: Given:

$$P = \{1, 2\}.$$

First, find the total number of elements

$$n(P \times P \times P).$$

Then, compare.

$P = \{1, 2\}$ and $n(P) = 2$

$\therefore n(P \times P \times P) = 8$

But there are 4 elements

Therefore, False

40.If

$$A = \{1, 2, 3\}, B = \{3, 4\} \text{ and } C = \{4, 5, 6\},$$

then

$$(A \times B) \cup (A \times C) = \{(1,3), (1,4), (1,5), (1,6), (2,3), (2,4), (2,5), (2,6), (3,3), (3,4), (3,5), (3,6)\}$$

Ans: Given:

$A = \{1, 2, 3\}$,

$B = \{3, 4\}$,

$C = \{4, 5, 6\}$.

First, find

$$A \times B \text{ and } A \times C,$$

then find

$$(A \times B) \cup (A \times C).$$

$A \times B = \{(1,3), (1,4), (2,3), (2,4), (3,3), (3,4)\}$,

$A \times C = \{(1,4), (1,5), (1,6), (2,4), (2,5), (2,6), (3,4), (3,5), (3,6)\}$,

$(A \times B) \cup (A \times C) = \{(1,3), (1,4), (1,5), (1,6), (2,3), (2,4), (2,5), (2,6), (3,3),$

$(3,4), (3,5), (3,6)\}$.

Therefore, True.