# SOLUTIONS

## Single Type

1. (D)

Let P(n) = $7^{2n} + 2^{3n-3}.3^{n-1} + n^2 - 3n + 2$

$P(1) = 50$

$P(2) = 2425$

$P(3) = 7^6 + 2^6.3^2 + 2 = 118227$

2. (C)

It $S_1$ is true, then $p(A \cap (B \cup C)) = P(A)P(B \cup C)$

$P(A \cap (B \cup C) = P((A \cap B) \cup (A \cap C))$

$= P(A \cap B) + P(A \cap C) - P((A \cap B) \cap (A \cap C)$

$= P(A)P(B) + P(A)P(C) - P(A \cap B \cap C)$

$= P(A)P(B) + P(A)P(C) - P(A)P(B)P(C)$

$= P(A)[P(B) + P(C) - P(B)P(C)]$

$= P(A)[P(B) + P(C) - P(B \cap C)] = P(A)P(B \cup C)$

It $S_2$ is true, then $P(A \cap (B \cap C) = P(A)P(B \cap C)$

$P(A \cap (B \cap C)) = P((A \cap B) \cap (A \cap C))$

$= P(A \cap B \cap C) = P(A)P(B)P(C) = P(A)P(B \cap C)$

3. (C)

For n = 1, the given expression is zero and hence the statement is true for n = 1

Suppose the statement is true for some m

$\therefore x(x^{m-1} - ma^{m-1}) + (m-1)a^m = (x-a)^2 Q$

$\therefore$ For n = m + 1, $\left(x^{m}-(m+1)a^{m}\right)+ma^{m+1}$

$= (x-a)^2 xQ + ma^{m-1}\left(x^2-2ax+a^2\right)$

$= (x-a)^2[xQ + ma^{m-1}]$

$\therefore$ By induction, the statement is true for each n.

4. (D)

Let P(n) = $7^{2n}+2^{3n-3}.3^{n-1}+n^2-3n+2$

$P(1)=50$

$P(2)=2425$

5. (A)

Putting n = 1 in $7^{2n}+2^{3n-3}.3^{n-1}$

then, $7^{2\times1}+2^{3\times1-3}.3^{1-1}$

$= 7^2+2^0.3^0=49+1=50$ ......(i)

Also, n = 2

$7^{2\times2}+2^{3\times2-3}.3^{2-1} = 2401+24=2425$ ......(ii)

From (i) and (ii) it is always divisible by 25.

6. (D)

$P(n)=n^2+n$. It is always odd (statement) but square of any odd number is always odd and also, sum of two odd number is always even. So for no any 'n' for which this statement is true.

7. (D)

Odd number (a) $n^p - n$ is divisible by p for any natural number greater than 1. It is Fermet's theorem.

Trick: Let n = 4 and p = 2

8. (C)

$2^4 \equiv 1$ (mod 5); $\Rightarrow (2^4)^{75} \equiv (1)^{75}$ (mod5)

i.e. $2^{300} \equiv 1$ (mod 5) $\Rightarrow 2^{300} \times 2 \equiv (1.2)$ (mod 5)

$\Rightarrow 2^{301} \equiv 2$ (mod 5), $\therefore$ Least positive remainder is 2.

9. (C)

$10^n + 3(4^{n+2}) + 5$

Taking n = 2; $10^2 + 3 \times 4^4 + 5$

= 100 + 768 + 5 = 873

Therefore this is divisible by 9.

10. (A)

$S_n = \frac{n(n+1)^2}{2}$ when n is even.

When n is odd,

Sum $= \left[1^2 + 2.2^2 + 2.3^2 + .... + 2(n-1)^2\right] + 2n^2$

In this case (n – 1) is even so the sum in the above bracket can be obtained by replacing n by (n – 1) in given result of $S_n$. Hence required sum $= \frac{(n-1)n^2}{2} + 2n^2 = \frac{n^2(n+3)}{2}$

11. (B)

$$A^2 = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} = \begin{pmatrix} \cos^2\theta - \sin^2\theta & 2\sin\theta\cos\theta \\ -2\sin\theta\cos\theta & \cos^2\theta - \sin^2\theta \end{pmatrix} = \begin{pmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{pmatrix} \text{etc.}$$

12. (C)

$P(n) = n^2 - n + 41$

$P(2) = 2^2 - 2 + 41 = 43$ (Prime true)

$P(3) = 3^2 - 3 + 41 = 47$ (Prime true)

$P(41) = 41^2 - 41 + 41 = (41)^2$ is Prime (false)

13. (C)

$P(n) = 9^n - 8^n$

$P(1) = 9 - 8 = 1$

$P(1) - 1 = 0$ which is divisible by 8

$\therefore P(1) = 1$ is the remainder, when $P(n)$ is divided by 8.

$P(2) = 9^2 - 8^2$

= 17 = 16 +1

Remainder is 1, when divided by 8.

14. (B)

$P(n) = a^n + b^n \forall n \in N$

n = 1

$\therefore$ P(1) = a + b which is divisible by a + b

n = 2

$\therefore$ $P(2) = a^2 + b^2$ not divisible by a + b

n = 3

$\therefore \quad P(3) = a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

which is divisible by a + b.

With the help of induction we conclude that P(n) will be divisible by a + b if n is odd.

15. (A)

Let $P(n) = 3^{2^n} - 1$

$P(1) = 3^{2^1} - 1 = 8 = 1.8$ is divisible by $2^3$.

$P(2) = 3^{2^2} - 1 = 80 = 5.2^4$ is divisible by $2^4$ .

13,14,15. Verification

16. (C)

$P(n) : n! > 2^{n-1}$

$P(3) : 3! > 2^2 \Rightarrow P(k): k! > 2^{k-1}$ is true for $k > 3$.

Now $P(k + 1) : (k + 1)! = (k + 1) . k!$

$> (k + 1) . 2^{k-1}$

$> 2^k$ as $k + 1 > 2$

So, $P(k) \Rightarrow P(k + 1)$ is true

Hence, $P(n): n! > 2^{n-1}$ is true for all $n > 3$.

17. (C)

$P(n) = 2. 4^{2n+1} + 3^{3n+1}$

$P(1) = 2.4^3 + 3^4 = 209$, divisible by 11.

$\Rightarrow P(k) = 2. 4^{2k+1} + 3^{3k+1}$ is divisible by 11. _____ (i)

Now $P(k + 1) = 2.\ 4^{2k+3} + 3^{3k+4}$

$\Rightarrow 2.\ 4^{2k+1}\ .\ 4^2 + 3^{3k+1}\ .27$

$\Rightarrow 2.\ 4^2(2.4^{2k+1} + 3^{3k+1}) + 11.3^{3k+1}$

$\Rightarrow$ P (k + 1) is divisible by 11, using (i)

$\Rightarrow$ so $P(n) = 2.4^{2n+1} + 3^{3n+1}$ is divisible by 11 for $n \geq 1$.

18. (B)

Let S(k) be true. Then,

$1 + 3 + 5 + ... (2k - 1) = 3 + k^2$ ...(1)

Now,

$S(k + 1) = 1 + 3 + 5 + .... + (2k - 1) + (2k + 1)$

$= (3 + k^2) + 2k + 1 = 3 + (k + 1)^2$ Using (1)

$\Rightarrow$ S (k + 1) is true.

Hence, option (B) is correct.

19. (C)

P(n) : 3 + 13 + 29 + 51 + 79 + .................... to n terms

P(1) = 3

$P(1) = 1^3 + 2.1^2$

again $P(2) = 16 = 2^3 + 2.2^2$

So, $P(n) = n^3 + 2.n^2$

20. (D)

P(n) : $\frac{n^5}{5}+\frac{n^3}{3}+\frac{7}{15^n}$

P(1) : $\frac{1}{5}+\frac{1}{3}+\frac{7}{15}=1$

P(k + 1) $=\frac{(k+1)^5}{5}+\frac{(k+1)^3}{3}+\frac{7}{15(k+1)}=\frac{3(k+1)^6+5(k+1)^4+7}{15(k+1)}$

$3(k+1)^6+5(k+1)^4+7$ is not always divisible by $15(k+1)$ i.e so, P(k + 1) is rational number

## Integer Type

21. (1)

$P(n)=5^n-2^n$

$n=1$

$\therefore P(5)=5^5-2^5$

$= 3125 - 32$

$= 3093$

$= 3 \times 1031$

In this case $\lambda=1031$

Similarly we can check the result for other cases and find that the least value of $\lambda$ and n is 1.

22. (24)

Since product of any r consecutive integers is divisible by r! and not divisible by r+1!.

So given product of 4 consecutive integers is divisible by 4! or 24.

23. (9)

Let three consecutive natural numbers are n, n+1, n+2, P(n) $= (n)^3 + (n+1)^3 + (n+2)^3$

$P(1) = 1^3 + 2^3 + 3^3 = 36$, which is divisible by 2 and 9

$P(2) = (2)^3 + (3)^3 + (4)^3 = 99$, which is divisible by 9 (not by 2).

Hence P(n) is divisible 9 $\forall\, n \in N$.

24. (6)

Let n is a positive integer.

$P(n) = n^3 - n$

$P(1) = 0$, which is divisible by for all $n \in N$

$P(2) = 6$, which is divisible by 6 (not by 4 and 9)

25. (5)

Let $P(n) = 10^n + 3.4^{n+2} + \lambda$ is divisible by 9 $\forall\, n \in N$

$P(1) = 10 + 3.4^3 + \lambda = 202 + \lambda = 207 + (\lambda - 5)$

Which is divisible by 9 if $\lambda = 5$

26. (8)

$5^{99} = (5)(5^2)^{49} = 5(25)^{49} = 5(26-1)^{49}$

$= 5 \times (26) \times (\text{Positive terms}) - 5$, So when it is divided by 13 it gives the remainder – 5 or (13 – 5) i.e., 8.

27. (6)

$n(n^2-1)=(n-1)(n)(n+1)$

It is product of three consecutive natural numbers, so according to Langrange's theorem it is divisible by 3 ! i.e., 6.

28. (8)

Let m = 2k + 1

n = 2k –1 as n < m.

$m^2 - n^2 = (2k + 1)^2 - (2k - 1)^2$

$= 4k^2 + 1 + 4k - 2k^2 - 1 + 4k$

P(k) = 8k

P(1) is divisible by 8, P(2) is divisible by 8

P(k) is divisible by 8

29. (64)

P(n) : $49^n + 16n - 1$

P(1) : 49 + 16 – 1, divisible by 64, 16, 8, 4

$P(n + 1) = 49^{n+1} + 16n + 16 - 1$

$= 49^n.49 + 16n + 16 - 1 + 49.\ 16n - 49 - 49.16n + 49$

$= 49(49^n + 16n - 1) - 48.16n + 64$

$= 49(49^n + 16n - 1) + 64(1 - 12n)$

P(n + 1), is divisible by 64,

So, P(n) is divisible by 64

30. (120)

Product of r consecutive integers is divisible by r!.

So given expression is divisible by 5! i.e. 1204