MATHEMATICAL INDUCTION

Single Type

1. If $n \in N$, then $7^{2n} + 2^{3n-3}.3^{n-1} + n^2 - 3n + 2$ is always divisible by

   (A) 25 (B) 35
   (C) 45 (D) none of these

2. Given that A, B, C are three mutually independent events. Consider the two statements

   $S_1 : A$ and $B \cup C$ are independent

   $S_2 : A$ and $B \cap C$ are independent.

   Then

   (A) only S_2 is true
   (B) S_1 may be true and S_2 must be true
   (C) both S_1 and S_2 are true
   (D) none of S_1 and S_2 are true

3. $x(x^{n-1} - na^{n-1}) + a^n(n-1)$ is divisible by $(x-a)^2$ for

   (A) $n > 1$ (B) $n > 2$
   (C) all $n \in N$ (D) none of these

4. If $n \in N$, then $7^{2n} + 2^{3n-3}.3^{n-1} + n^2 - 3n + 2$ is always divisible by

   (A) 25 (B) 35
   (C) 45 (D) none of these

5. $P(3) = 7^6 + 2^6 .3^2 + 2 = 118227$. If $n \in N$, then $7^{2n} + 2^{3n-3}.3^{n-1}$ is always divisible by

(A) 25 (B) 35

(C) 45 (D) None of these

6. Let P(n) denote the statement that $n^2 + n$ is odd. It is seen that $P(n) \Rightarrow P(n+1)$, P_n is true for all

(A) $n > 1$ (B) n

(C) $n > 2$ (D) None of these

7. If p is a prime number, then $n^p - n$ is divisible by p when n is a

(A) Natural number greater than 1

(B) Irrational number

(C) Complex number

(D) Odd number

8. Then $(4)^2 - 4 = 16 - 4 = 12$, it is divisible by 2. So, it is true for any natural number greater than 1. When 2^{301} is divided by 5, the least positive remainder is

(A) 4 (B) 8

(C) 2 (D) 6

9. $10^n + 3(4^{n+2}) + 5$ is divisible by $(n \in N)$

(A) 7 (B) 5

(C) 9 (D) 17

10. The sum of first n terms of the series $1^2 + 2.2^2 + 2.3^2 + ...2.4^2 + ...$ is $\frac{n(n+1)^2}{2}$ when n is even. If n is odd, the sum is

(A) $\frac{n^2(n+3)}{2}$ (B) $\frac{n(4n^2-1)}{3}$

(C) $\frac{3n(n+1)^2}{8}$ (D) $\frac{3n(n+1)}{2}$

11. If $A = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$, then for any $n \in N, A^n$ equals

(A) $\begin{pmatrix} \cos^n\theta & \sin^n\theta \\ -\sin^n\theta & \cos^n\theta \end{pmatrix}$ (B) $\begin{pmatrix} \cos n\theta & \sin n\theta \\ -\sin n\theta & \cos n\theta \end{pmatrix}$

(C) $\begin{pmatrix} n\cos\theta & n\sin\theta \\ -n\sin\theta & n\cos\theta \end{pmatrix}$ (D) None of these

12. Let be the statement $n^2 - n + 41$ is prime, then which of the following is not true

(A) P(2) (B) P(3)

(C) P(41) (D) P(5)

13. The statement $P(n) = 9^n - 8^n$, when divided 8, always leaves the remainder

(A) 2 (B) 3

(C) 1 (D) 7

14. Let $P(n): a^n + b^n$ such that a, b are even, then $P(n)$ will be divisible by $a+b$ if

(A) $n > 1$
(B) $\forall n$ is odd
(C) $\forall n$ is even
(D) $n > 2$

15. For every integer $n \geq 1 (3^{2^n} - 1)$, is divisible by

(A) 2^{n+2}
(B) 2^{n+1}
(C) 2^{n+3}
(D) 2^n

16. The inequality $n! > 2^{n-1}$ is true for

(A) $n > 2$
(B) $n \in N$
(C) $n > 3$
(D) $n > 1$

17. For all $n \in N$, $2.4^{2n+1} + 3^{3n+1}$ is divisible by

(A) 7
(B) 5
(C) 11
(D) 209

18. Let $S(k) = 1 + 3 + 5 + ... (2k - 1) = 3 + k^2$. Then which of the following is true?

(A) Principle of mathematical induction can be used to prove the formula
(B) $S(k) \Rightarrow S(k + 1)$
(C) $S(k) \Rightarrow S(k^2 + 1)$
(D) S(1) is correct

19. 3 + 13 + 29 + 51 + 79 + ... to n terms

(A) $2n^2 + 7n^3$ (B) $n^2 + 5n^3$

(C) $n^3 + 2n^2$ (D) none of these

20. For all $n \in N, \frac{n^5}{5} + \frac{n^3}{3} + \frac{7}{15n}$ is

(A) an integer (B) a natural number

(C) a positive fraction (D) rational number

Integer Type

21. Let $P(n) = 5^n - 2^n$, P(n) is divisible by 3λ where λ and n both are odd positive integers then the least value of n and λ will be

22. The greatest positive integer. Which divides (n + 16) (n + 17) (n + 18) (n + 19), for all $n \in N$, is-

23. The sum of the cubes of three consecutive natural numbers is divisible by-

24. The difference between an +ve integer and its cube is divisible by-

25. If $10^n + 3.4^{n+2} + \lambda$ is exactly divisible by 9 for all $n \in N$, then the least positive integral value of λ is-

26. The remainder when 5^{99} is divided by 13 is

27. For every natural number n, $n(n^2 - 1)$ is divisible by

28. If m, n are any two odd positive integer with n < m then the largest positive integers which divides all the numbers of the type $m^2 - n^2$ is:

29. For all $n \in N$, $49^n + 16n - 1$ is divisible by

30. The greatest positive integer, which divides (n + 2) (n + 3) (n + 4) (n + 5) (n + 6) for all $n \in N$ is