Q1 If $\log_5 120+(x-3)-2\log_5(1-5^{x-3})=-\log_5(0.2-5^{x-4})$, then x is

(a) 1 (b) 2

(c) 3 (d) 4

Ans 1

$$1-5^{x-3}>0$$
$$\Rightarrow 5^{x-3}<1$$
$$\Rightarrow x-3<0$$
$$\Rightarrow x<3$$

And

$$0.2-5^{x-4}>0$$
$$\Rightarrow 5^{x-4}<0.2$$
$$\Rightarrow 5^{x-4}<5^{-1}$$
$$\Rightarrow x-4<-1$$
$$\Rightarrow x<3 \quad ......(1)$$

$$\log_5 120+(x-3)-2\log_5(1-5^{x-3})=-\log_5(0.2-5^{x-4})$$

$$\Rightarrow \log_5(24\times5)+(x-3)-2\log_5(1-5^{x-3})=-\log_5(0.2-5^{x-4})$$
$$\Rightarrow \log_5 24+1+(x-3)-2\log_5(1-5^{x-3})=-\log_5\{0.2(1-5^{x-3})\}$$
$$\Rightarrow \log_5 24+x-2-2\log_5(1-5^{x-3})=-\log(0.2)-\log_5(1-5^{x-3})$$
$$\Rightarrow \log_5 24+x-2-\log_5(1-5^{x-3})=1$$
$$\Rightarrow \log_5\left\{\frac{1-5^{x-3}}{24}\right\}=x-3$$
$$\Rightarrow \frac{1-5^{x-3}}{24}=5^{x-3}$$
$$\Rightarrow 1=25.5^{x-3}$$
$$\Rightarrow 1=5^{x-1}$$
$$\Rightarrow x-1=0$$
$$\Rightarrow x=1$$

Hence, the correct option is (a).

Q 2. If $\log_{10}2, \log_{10}(2^x+1), \log_{10}(2^x+3)$ are in AP, then

(a) $x=0$ (b) $x=1$

(c) $x=\log_{10}2$ (d) $x=\frac{1}{2}\log_{10}5$

Ans 2 As $\log_{10}2, \log_{10}(2^x+1), \log_{10}(2^x+3)$ are in AP

Therefore, 2, 2^x+1, 2^x+3 are in GP

$$\Rightarrow (2^x+1)^2=2(2^x+3)$$
$$\Rightarrow 2^{2x}=5$$
$$\Rightarrow 2x=\log_2 5$$

Q3 If $\log_2 x+\log_2 y\ge 6$, then the least value of $x+y$ is

(a) 4 (b) 8

(c) 16 (d) 32

Ans 3 $\log_2 x+\log_2 y\ge 6$

$$\log_2 x+\log_2 y\ge 6$$
$$\Rightarrow \log_2 xy\ge 6$$
$$\Rightarrow xy\ge 2^6$$
$$\Rightarrow \sqrt{xy}\ge 2^3$$

As,

$$\frac{x+y}{2}\ge\sqrt{xy}$$
$$\Rightarrow x+y\ge 2\sqrt{xy}\ge 16 \quad (\because \text{AM}\ge\text{GM})$$
$$\Rightarrow x+y\ge 16$$

Hence, the correct option is (c).

Q4. $7\log\left(\frac{16}{15}\right)+5\log\left(\frac{25}{24}\right)+3\log\left(\frac{81}{80}\right)$ is equal to

(a) 0 (b) 1

(c) log 2 (d) log 3

Ans 4

$$7\log\left(\frac{16}{15}\right)+5\log\left(\frac{25}{24}\right)+3\log\left(\frac{81}{80}\right)$$
$$=7\log\left(\frac{2^4}{5\times3}\right)+5\log\left(\frac{5^2}{2^3\times3}\right)+3\log\left(\frac{3^4}{2^4\times5}\right)$$

$$=7\{4\log2-\log5-\log3\}+5\{2\log5-3\log2-\log3\}+3\{4\log3-4\log2-\log5\}$$
$$=\log 2$$

Hence, the correct option is (c).

Q5. The value of $\frac{1}{\log_2 n}+\frac{1}{\log_3 n}+....+\frac{1}{\log_{43} n}$ is

(a) $\frac{1}{\log_{43!} n}$ (b) $\frac{1}{\log_{43} n}$

(c) $\frac{1}{\log_{42} n}$ (d) $\frac{1}{\log_{43} n!}$

Ans 5

$$\frac{1}{\log_2 n}+\frac{1}{\log_3 n}+....+\frac{1}{\log_{43} n}$$
$$=\log_n 2+\log_n 3+....+\log_n 43$$
$$=\log_n(2\cdot3\cdot...43)$$
$$=\log_n 43!$$
$$=\frac{1}{\log_{43!} n}$$

Hence, the correct option is (a).

Q6. $\log_7\log_7\sqrt{7\sqrt{(7\sqrt{7})}}$ is equal to

(a) $3\log_2 7$ (b) $3\log_7 7$

(c) $1-3\log_7 2$ (d) $1-3\log_2 7$

Ans 6

$$\log_7\log_7 7^{\frac{1}{2}+\frac{1}{4}+\frac{1}{8}}=\log_7\left(\frac{1}{2}+\frac{1}{4}+\frac{1}{8}\right)=\log_7\left(\frac{7}{8}\right)$$
$$=1-\log_7 8$$
$$=1-3\log_7 2$$

Hence, the correct option is (c).

Q7 . If *x, y, z* are in GP and $a^x=b^y=c^z$, then

(a) $\log_b a=\log_c b$ (b) $\log_c b=\log_a c$

(c) $\log_a c=\log_b a$ (d) $\log_a b=2\log_a c$

Ans 7

$$a^x=b^y=c^z$$
$$\Rightarrow \quad x\log a=y\log b=z\log c$$
$$\therefore \quad \frac{y}{x}=\frac{z}{y}\Rightarrow\frac{\log a}{\log b}=\frac{\log b}{\log c}$$
$$\Rightarrow \quad \log_b a=\log_c b$$

Hence, the correct option is (a).

Q 8. If $y=a^{\frac{1}{1-\log_a x}}$ and $z=a^{\frac{1}{1-\log_a y}}$, then x is equal to

(a) $a^{\frac{1}{1+\log_a x}}$ (b) $a^{\frac{1}{2+\log_a z}}$

(c) $a^{\frac{1}{1-\log_a z}}$ (d) $a^{\frac{1}{2-\log_a z}}$

Ans 8

$$\log_a y=\frac{1}{1-\log_a x}$$
$$\therefore \quad 1-\log_a y=1-\frac{1}{1-\log_a x}$$
$$=\frac{-\log_a x}{1-\log_a x}$$
$$\text{or} \quad \frac{1}{1-\log_a y}=\frac{1-\log_a x}{-\log_a x}$$
$$\text{But} \quad z=a\frac{1}{1-\log_a y}$$

$$\Rightarrow \quad \log_a z=\frac{1}{1-\log_a y}=-\frac{1}{\log x}+1$$
$$\Rightarrow \quad \frac{1}{\log_a x}=1-\log_a z$$
$$\log_a x=\frac{1}{1-\log_a z}$$
$$\therefore \quad x=a^{\frac{1}{1-\log_a z}}$$

Hence, the correct option is (c).

Q 9. If $\frac{1}{\log_3\pi}+\frac{1}{\log_4\pi}>x$, then x be

(a) 3 (b) 2

(c) π (d) none of these

Ans 9

$$\frac{1}{\log_3\pi}+\frac{1}{\log_4\pi}>x$$
$$\Rightarrow \quad \log_\pi 3+\log_\pi 4>x$$
$$\Rightarrow \quad \log_\pi 12>x\Rightarrow 12>\pi^x$$
$$\therefore \quad x=2$$

Hence, the correc t option is (b).

Q10. The solution set of the equation

$\log_x 2\log_{2x}2=\log_{4x}2$ is

(a) $\{2^{-\sqrt{2}},2^{\sqrt{2}}\}$ (b) $\{1/2,2\}$

(c) $\{1/4,2^2\}$ (d) none of these

Ans 10

$$\because \quad \log_x 2\log_{2x}2=\log_{4x}2$$
$$\because \quad x>0, 2x\ne0 \text{ and } 4x>0 \text{ and}$$
$$x\ne1, 2x\ne1, 4x\ne1$$
$$\Rightarrow \quad x>0 \text{ and } x\ne1,\frac{1}{2},\frac{1}{4}$$

Then , $\frac{1}{\log_2 x}\cdot\frac{1}{\log_2 2x}=\frac{1}{\log_2 4x}$

$$\Rightarrow \quad \log_2 x.\log_2 2x=\log_2 4x$$
$$\Rightarrow \quad \log_2 x.(1+\log_2 x)=(2+\log_2 x)$$
$$\Rightarrow \quad (\log_2 x)^2=2$$
$$\Rightarrow \quad \log_2 x=\pm\sqrt{2}$$
$$\therefore \quad x=2^{\pm\sqrt{2}}$$
$$\therefore \quad x=\{2^{-\sqrt{2}},2^{\sqrt{2}}\}$$

Hence, the correct option is (a).