Related Problems with Solutions

Problem 3:

Question 9:

A diet is to contain at least 80 units of vitamin A and 100 units of minerals. Two foods $F_1$and F_2 are available. Food F_1 costs Rs 4 per unit food and F_2 costs Rs 6 per unit. One unit of food F_1 contains 3 units of vitamin A and 4 units of minerals. One unit of food F_2 contains 6 units of vitamin A and 3 units of minerals. Formulate this as a linear programming problem. Find the minimum cost for diet that consists of mixture of these two foods and also meets the minimal nutritional requirements?

Answer

Let the diet contain x units of food F_1 and y units of food F_2. Therefore,

$x \geq 0$ and $y \geq 0$

The given information can be complied in a table as follows.

	Vitamin A (units)	**Mineral (units)**	**Cost per unit (Rs)**
Food F_1 (x)	3	4	4
Food F_2 (y)	6	3	6
Requirement	80	100	

The cost of food F_1 is Rs 4 per unit and of Food F_2 is Rs 6 per unit. Therefore, the constraints are

$3x + 6y \geq 80$

$4x + 3y \geq 100$

$x, y \geq 0$

Total cost of the diet, $Z = 4x + 6y$

The mathematical formulation of the given problem is

Minimise $Z = 4x + 6y$ … (1)

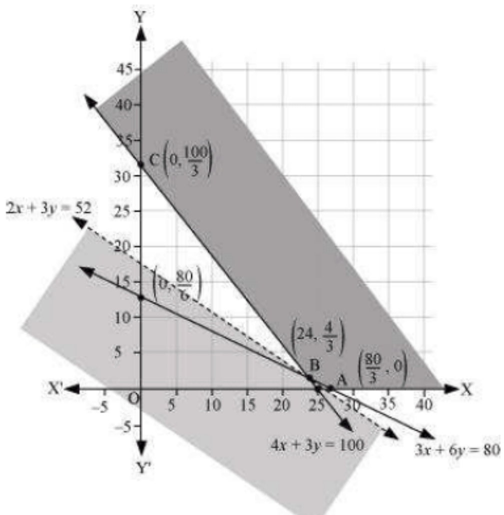
subject to the constraints,

$3x + 6y \geq 80$ … (2)

$4x + 3y \geq 100$ … (3)

$x, y \geq 0$ … (4)

The feasible region determined by the constraints is as follows.



It can be seen that the feasible region is unbounded.

The corner points of the feasible region are $A\left(\frac{8}{3}, 0\right)$, $B\left(2, \frac{1}{2}\right)$, and $C\left(0, \frac{11}{2}\right)$.

The corner points are $A\left(\frac{80}{3}, 0\right)$, $B\left(24, \frac{4}{3}\right)$, and $C\left(0, \frac{100}{3}\right)$.

The values of Z at these corner points are as follows.

Corner point	**Z = 4*x* + 6*y***	
$A\left(\frac{80}{3}, 0\right)$	$\frac{320}{3} = 106.67$	
$B\left(24, \frac{4}{3}\right)$	104	→ Minimum
$C\left(0, \frac{100}{3}\right)$	200	

As the feasible region is unbounded, therefore, 104 may or may not be the minimum value of Z.

For this, we draw a graph of the inequality, $4x + 6y < 104$ or $2x + 3y < 52$, and check whether the resulting half plane has points in common with the feasible region or not.

It can be seen that the feasible region has no common point with $2x + 3y < 52$

Therefore, the minimum cost of the mixture will be Rs 104.