2. Minimise $Z = -3x + 4y$
   subject to $x + 2y \leq 8$, $3x + 2y \leq 12$, $x \geq 0$, $y \geq 0$.
3. Maximise $Z = 5x + 3y$
   subject to $3x + 5y \leq 15$, $5x + 2y \leq 10$, $x \geq 0$, $y \geq 0$.
4. Minimise $Z = 3x + 5y$
   such that $x + 3y \geq 3$, $x + y \geq 2$, $x, y \geq 0$.
5. Maximise $Z = 3x + 2y$
   subject to $x + 2y \leq 10$, $3x + y \leq 15$, $x, y \geq 0$.
6. Minimise $Z = x + 2y$
   subject to $2x + y \geq 3$, $x + 2y \geq 6$, $x, y \geq 0$.

Show that the minimum of Z occurs at more than two points.

7. Minimise and Maximise $Z = 5x + 10y$
   subject to $x + 2y \leq 120$, $x + y \geq 60$, $x - 2y \geq 0$, $x, y \geq 0$.
8. Minimise and Maximise $Z = x + 2y$
   subject to $x + 2y \geq 100$, $2x - y \leq 0$, $2x + y \leq 200$; $x, y \geq 0$.
9. Maximise $Z = -x + 2y$, subject to the constraints:
   $x \geq 3$, $x + y \geq 5$, $x + 2y \geq 6$, $y \geq 0$.
10. Maximise $Z = x + y$, subject to $x - y \leq -1$, $-x + y \leq 0$, $x, y \geq 0$.

12.3 Different Types of Linear Programming Problems

A few important linear programming problems are listed below:

1. **Manufacturing problems** In these problems, we determine the number of units of different products which should be produced and sold by a firm when each product requires a fixed manpower, machine hours, labour hour per unit of product, warehouse space per unit of the output etc., in order to make maximum profit.
2. **Diet problems** In these problems, we determine the amount of different kinds of constituents/nutrients which should be included in a diet so as to minimise the cost of the desired diet such that it contains a certain minimum amount of each constituent/nutrients.
3. **Transportation problems** In these problems, we determine a transportation schedule in order to find the cheapest way of transporting a product from plants/factories situated at different locations to different markets.

Let us now solve some of these types of linear programming problems:

Example 6 (Diet problem): A dietician wishes to mix two types of foods in such a way that vitamin contents of the mixture contain atleast 8 units of vitamin A and 10 units of vitamin C. Food 'I' contains 2 units/kg of vitamin A and 1 unit/kg of vitamin C. Food 'II' contains 1 unit/kg of vitamin A and 2 units/kg of vitamin C. It costs Rs 50 per kg to purchase Food 'I' and Rs 70 per kg to purchase Food 'II'. Formulate this problem as a linear programming problem to minimise the cost of such a mixture.

Solution Let the mixture contain x kg of Food 'I' and y kg of Food 'II'. Clearly, $x \geq 0$, $y \geq 0$. We make the following table from the given data:

Resources	Food I (x)	Food II (y)	Requirement
Vitamin A (units/kg)	2	1	8
Vitamin C (units/kg)	1	2	10
Cost (Rs/kg)	50	70	

Since the mixture must contain at least 8 units of vitamin A and 10 units of vitamin C, we have the constraints:

$$2x + y \geq 8$$

$$x + 2y \geq 10$$

Total cost Z of purchasing x kg of food 'I' and y kg of Food 'II' is

$$Z = 50x + 70y$$

Hence, the mathematical formulation of the problem is:

Minimise $Z = 50x + 70y$ ... (1)

subject to the constraints:

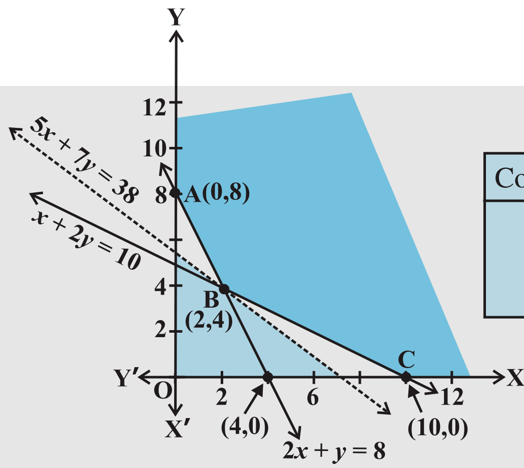
$2x + y \geq 8$ ... (2)

$x + 2y \geq 10$ ... (3)

$x, y \geq 0$ ... (4)

Let us graph the inequalities (2) to (4). The feasible region determined by the system is shown in the Fig 12.7. Here again, observe that the feasible region is **unbounded**.

Let us evaluate Z at the corner points A(0,8), B(2,4) and C(10,0).



Corner Point	$Z = 50x + 70y$
(0,8)	560
(2,4)	**380** ← Minimum
(10,0)	500

Fig 12.7

In the table, we find that smallest value of Z is 380 at the point (2,4). Can we say that the minimum value of Z is 380? Remember that the feasible region is unbounded. Therefore, we have to draw the graph of the inequality

$$50x + 70y < 380 \text{ i.e., } \quad 5x + 7y < 38$$

to check whether the resulting open half plane has any point common with the feasible region. From the Fig 12.7, we see that it has no points in common.

Thus, the minimum value of Z is 380 attained at the point (2, 4). Hence, the optimal mixing strategy for the dietician would be to mix 2 kg of Food 'I' and 4 kg of Food 'II', and with this strategy, the minimum cost of the mixture will be Rs 380.

Example 7 (Allocation problem) A cooperative society of farmers has 50 hectare of land to grow two crops X and Y. The profit from crops X and Y per hectare are estimated as Rs 10,500 and Rs 9,000 respectively. To control weeds, a liquid herbicide has to be used for crops X and Y at rates of 20 litres and 10 litres per hectare. Further, no more than 800 litres of herbicide should be used in order to protect fish and wild life using a pond which collects drainage from this land. How much land should be allocated to each crop so as to maximise the total profit of the society?

Solution Let x hectare of land be allocated to crop X and y hectare to crop Y. Obviously, $x \geq 0$, $y \geq 0$.

Profit per hectare on crop X = Rs 10500

Profit per hectare on crop Y = Rs 9000

Therefore, total profit = Rs $(10500x + 9000y)$

The mathematical formulation of the problem is as follows:

Maximise $Z = 10500\,x + 9000\,y$

subject to the constraints:

$x + y \leq 50$ (constraint related to land) ... (1)

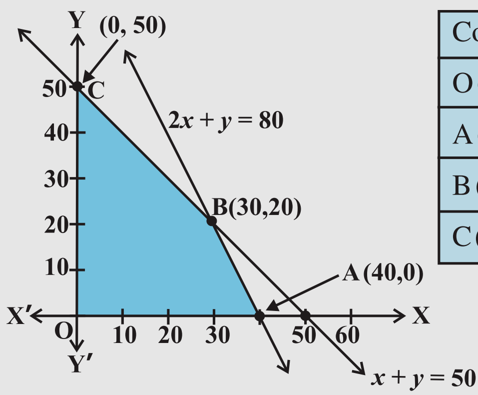
$20x + 10y \leq 800$ (constraint related to use of herbicide)

i.e. $2x + y \leq 80$ ... (2)

$x \geq 0,\ y \geq 0$ (non negative constraint) ... (3)

Let us draw the graph of the system of inequalities (1) to (3). The feasible region OABC is shown (shaded) in the Fig 12.8. Observe that the feasible region is **bounded**.

The coordinates of the corner points O, A, B and C are (0, 0), (40, 0), (30, 20) and (0, 50) respectively. Let us evaluate the objective function $Z = 10500\,x + 9000y$ at these vertices to find which one gives the maximum profit.



Corner Point	$Z = 10500x + 9000y$	
O(0, 0)	0	
A(40, 0)	420000	
B (30, 20)	**495000** ←	Maximum
C (0,50)	450000	

Fig 12.8

Hence, the society will get the maximum profit of Rs 4,95,000 by allocating 30 hectares for crop X and 20 hectares for crop Y.

Example 8 **(Manufacturing problem)** A manufacturing company makes two models A and B of a product. Each piece of Model A requires 9 labour hours for fabricating and 1 labour hour for finishing. Each piece of Model B requires 12 labour hours for fabricating and 3 labour hours for finishing. For fabricating and finishing, the maximum labour hours available are 180 and 30 respectively. The company makes a profit of Rs 8000 on each piece of model A and Rs 12000 on each piece of Model B. How many pieces of Model A and Model B should be manufactured per week to realise a maximum profit? What is the maximum profit per week?

Solution Suppose x is the number of pieces of Model A and y is the number of pieces of Model B. Then

$$\text{Total profit (in Rs)} = 8000\,x + 12000\,y$$

Let $$Z = 8000\,x + 12000\,y$$

We now have the following mathematical model for the given problem.

Maximise $Z = 8000\,x + 12000\,y$ ... (1)

subject to the constraints:

$9x + 12y \leq 180$ (Fabricating constraint)

i.e. $3x + 4y \leq 60$ ... (2)

$x + 3y \leq 30$ (Finishing constraint) ... (3)

$x \geq 0,\ y \geq 0$ (non-negative constraint) ... (4)

The feasible region (shaded) OABC determined by the linear inequalities (2) to (4) is shown in the Fig 12.9. Note that the feasible region is bounded.

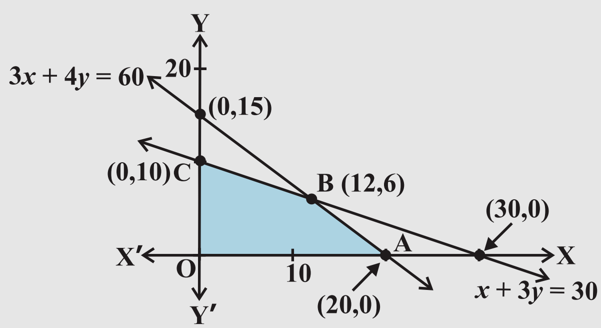


Fig 12.9

Let us evaluate the objective function Z at each corner point as shown below:

Corner Point	$Z = 8000\,x + 12000\,y$	
0 (0, 0)	0	
A (20, 0)	160000	
B (12, 6)	**168000** ⟵	Maximum
C (0, 10)	120000	

We find that maximum value of Z is 1,68,000 at B (12, 6). Hence, the company should produce 12 pieces of Model A and 6 pieces of Model B to realise maximum profit and maximum profit then will be Rs 1,68,000.