Previous Year CBSE Problems with Solutions

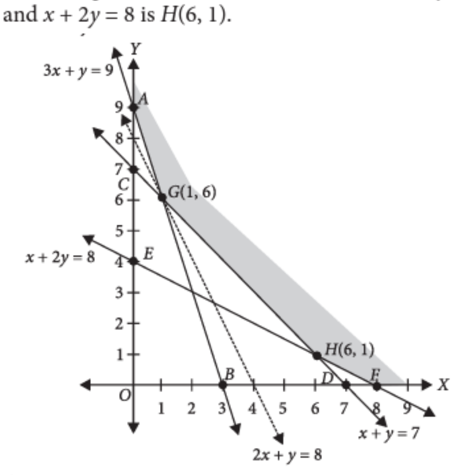
Problem 3: 5. In order to supplement daily diet, a person wishes to take X and Y tablets. The contents (in milligrams per tablet) of iron, calcium and vitamins in X and Y are given as below:

Tablets	Iron	Calcium	Vitamin
X	6	3	2
Y	2	3	4

The person needs to supplement at least 18 milligrams of iron, 21 milligrams of calcium and 16 milligrams of vitamins. The price of each tablet of X and Y is Rs. 2 and Rs. 1 respectively. How many tablets of each type should the person take in order to satisfy the above requirement at the minimum cost? Make an LPP and solve graphically. *(Foreign 2016)*

Solution:

5. Let the person takes x tablets of type X and y tablets of type Y.
According to the given conditions, we have
$6x + 2y \geq 18 \Rightarrow 3x + y \geq 9$
$3x + 3y \geq 21 \Rightarrow x + y \geq 7$
$2x + 4y \geq 16 \Rightarrow x + 2y \geq 8$
Let z be the total cost of tablets.
$\therefore \ z = 2x + y$
Hence, the given LPP is
Minimise $Z = 2x + y$
subject to the constraints
$3x + y \geq 9, x + y \geq 7, x + 2y \geq 8$ and $x, y \geq 0$
To solve graphically, we convert the inequations into equations.
$3x + y = 9, x + y = 7, x + 2y = 8, x = 0, y = 0$
The line $3x + y = 9$ meets the coordinate axes at $A(0, 9)$ and $B(3, 0)$. Similarly, $x + y = 7$ meets the coordinate axes at $C(0, 7)$ and $D(7, 0)$. Also, line $x + 2y = 8$ meets the coordinate axes at $E(0, 4)$, $F(8, 0)$
The point of intersection of the lines $3x + y = 9$ and $x + y = 7$ is $G(1, 6)$.
Also, the point of intersection of the lines $x + y = 7$ and $x + 2y = 8$ is $H(6, 1)$.



The shaded region $AGHF$ represents the feasible region of the given LPP. The corner points of the feasible region are $A(0, 9)$, $G(1, 6)$, $H(6, 1)$ and $F(8, 0)$.
The values of the objective function at these points are given in the following table :

Corner Points	Value of $Z = 2x + y$
$A(0, 9)$	$2 \times 0 + 9 = 9$
$G(1, 6)$	$2 \times 1 + 6 = 8$ (Minimum)
$H(6, 1)$	$2 \times 6 + 1 = 13$
$F(8, 0)$	$2 \times 8 + 0 = 16$

From the table, we find that 8 is the minimum value of Z at $G(1, 6)$. Since the region is unbounded we have to check that the inequality $2x + y < 8$ in open half plane has any point in common or not.
Since, it has no point in common. So, Z is minimum at $G(1, 6)$ and the minimum value of Z is 8.
Hence, the person should take 1 tablet of type X and 6 tablets of type Y in order to meet the requirements at the minimum cost.