Previous Year CBSE Problems with Solutions

Problem 2:

4. There are two types of fertilisers 'A' and 'B'. 'A' consists of 12% nitrogen and 5% phosphoric acid whereas 'B' consists of 4% nitrogen and 5% phosphoric acid. After testing the soil conditions, farmer finds that he needs at least 12 kg of nitrogen and 12 kg of phosphoric acid for his crops. If 'A' costs ₹ 10 per kg and 'B' costs ₹ 8 per kg, then graphically determine how much of each type of fertiliser should be used so that the nutrient requirements are met at a minimum cost. *(AI 2016)*

Solution:

4. Let the requirement of fertiliser A by the farmer be x kg and that of B be y kg.

	Fertiliser A	Fertiliser B	Minimum requirement (in kg)
Nitrogen (in %)	12	4	12
Phosphoric acid/(in %)	5	5	12
Cost (in ₹ kg)	10	8	

The inequations thus formed based on the given problem will be as follows:

$$\frac{12x}{100}+\frac{4y}{100}\geq 12 \Rightarrow 12x+4y\geq 1200 \Rightarrow 3x+y\geq 300$$

Also, $\frac{5x}{100}+\frac{5y}{100}\geq 12$

$\Rightarrow 5x+5y\geq 1200 \Rightarrow x+y\geq 240$

and $x\geq 0$, $y\geq 0$.

Let Z be the total cost of the fertilisers. Then
$Z = 10x + 8y$

The LPP can be stated mathematically as
Minimise $Z = 10x + 8y$
subject to constraints $3x + y \geq 300$, $x + y \geq 240$, $x \geq 0$, $y \geq 0$.

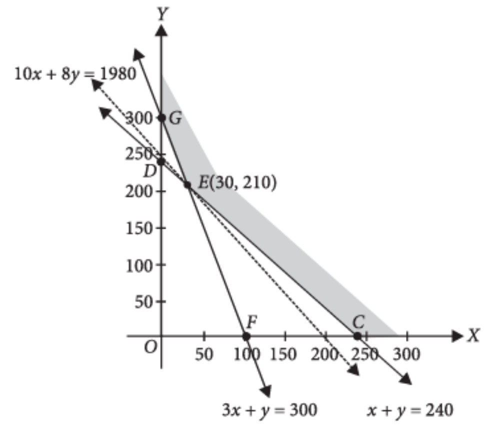
To solve the LPP graphically, we convert the inequations into equations to obtain the following lines:

$3x + y = 300$, $x + y = 240$, $x = 0$ and $y = 0$

Equation $3x + y = 300$ meets the coordinate axes at points $F(100, 0)$ and $G(0, 300)$

Equation $x + y = 240$ meets the coordinate axes at points $C(240, 0)$ and $D(0, 240)$.

The point of intersection of lines $3x + y = 300$ and $x + y = 240$ is $E(30, 210)$



The shaded region GEC represents the feasible region of given LPP and it is unbounded.

Corner points	Value of $Z = 10x + 8y$
$G(0, 300)$	2400
$C(240, 0)$	2400
$E(30, 210)$	1980 (Minimum)

From the table, we find that 1980 is the minimum value of z at $E(30, 210)$. Since the region is unbounded, we have check that the inequality $10x + 8y < 1980$ in open half plane has any point in common or not. Since, it has no point in common. So, the minimum value of Z is obtained at $E(30, 210)$ and the minimum value of Z is 1980.

So, the minimum requirement of fertiliser of type A will be 30 kg and that of type B will be 210 kg.