Exemplar problem

Example 10 **(Manufacturing problem)** A manufacturer has three machines I, II and III installed in his factory. Machines I and II are capable of being operated for at most 12 hours whereas machine III must be operated for atleast 5 hours a day. She produces only two items M and N each requiring the use of all the three machines.

The number of hours required for producing 1 unit of each of M and N on the three machines are given in the following table:

Items	**Number of hours required on machines**		
	I	II	III
M	1	2	1
N	2	1	1.25

She makes a profit of Rs 600 and Rs 400 on items M and N respectively. How many of each item should she produce so as to maximise her profit assuming that she can sell all the items that she produced? What will be the maximum profit?

Solution Let x and y be the number of items M and N respectively.
Total profit on the production = Rs $(600\,x + 400\,y)$
Mathematical formulation of the given problem is as follows:
Maximise $Z = 600\,x + 400\,y$
subject to the constraints:

$$x + 2y \leq 12 \text{ (constraint on Machine I)} \quad \ldots (1)$$

$$2x + y \leq 12 \text{ (constraint on Machine II)} \quad \ldots (2)$$

$$x + \frac{5}{4}\,y \geq 5 \text{ (constraint on Machine III)} \quad \ldots (3)$$

$$x \geq 0,\ y \geq 0 \quad \ldots (4)$$

Let us draw the graph of constraints (1) to (4). ABCDE is the feasible region (shaded) as shown in Fig 12.11 determined by the constraints (1) to (4). Observe that the feasible region is bounded, coordinates of the corner points A, B, C, D and E are (5, 0) (6, 0), (4, 4), (0, 6) and (0, 4) respectively.

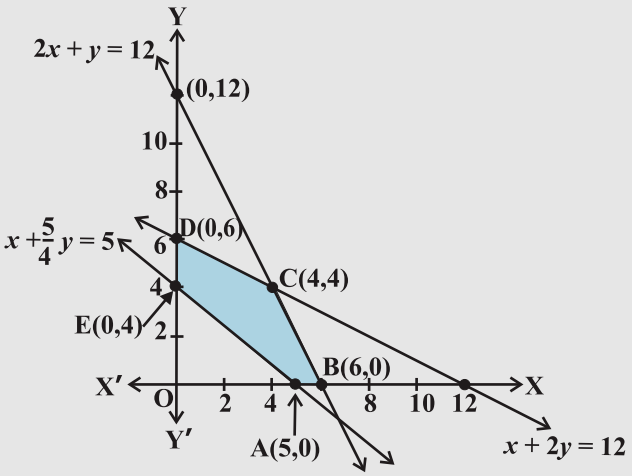

Fig 12.11

Let us evaluate Z = 600 x + 400 y at these corner points.

Corner point	Z = 600 x + 400 y	
(5, 0)	3000	
(6, 0)	3600	
(4, 4)	**4000** ←	Maximum
(0, 6)	2400	
(0, 4)	1600	

We see that the point (4, 4) is giving the maximum value of Z. Hence, the manufacturer has to produce 4 units of each item to get the maximum profit of Rs 4000.