Example 4 Determine graphically the minimum value of the objective function

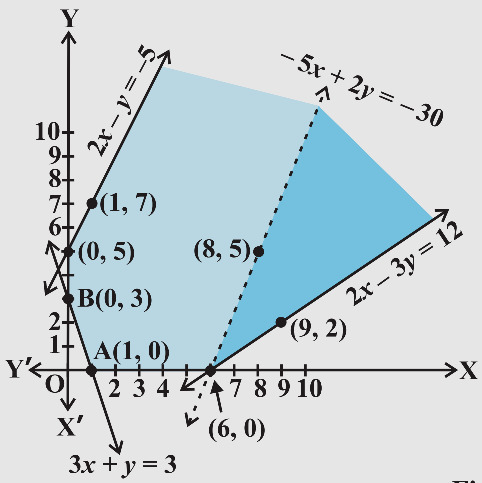
$$Z = -50x + 20y \quad \text{... (1)}$$

subject to the constraints:

$$2x - y \geq -5 \quad \text{... (2)}$$
$$3x + y \geq 3 \quad \text{... (3)}$$
$$2x - 3y \leq 12 \quad \text{... (4)}$$
$$x \geq 0, y \geq 0 \quad \text{... (5)}$$

Solution First of all, let us graph the feasible region of the system of inequalities (2) to (5). The feasible region (shaded) is shown in the Fig 12.5. Observe that the feasible region is **unbounded**.

We now evaluate Z at the corner points.



Corner Point	$Z = -50x + 20y$	
(0, 5)	100	
(0, 3)	**60**	
(1, 0)	**–50**	
(6, 0)	**– 300** ⟵	smallest

Fig 12.5

From this table, we find that – 300 is the smallest value of Z at the corner point (6, 0). Can we say that minimum value of Z is – 300? Note that if the region would have been bounded, this smallest value of Z is the minimum value of Z (Theorem 2). But here we see that the feasible region is unbounded. Therefore, – 300 may or may not be the minimum value of Z. To decide this issue, we graph the inequality

$$-50x + 20y < -300 \text{ (see Step 3(ii) of corner Point Method.)}$$

i.e., $$-5x + 2y < -30$$

and check whether the resulting open half plane has points in common with feasible region or not. If it has common points, then –300 will not be the minimum value of *Z*. Otherwise, –300 will be the minimum value of Z.

As shown in the Fig 12.5, it has common points. Therefore, Z = –50 x + 20 y has no minimum value subject to the given constraints.

In the above example, can you say whether z = – 50 x + 20 y has the maximum value 100 at (0,5)? For this, check whether the graph of – 50 x + 20 y > 100 has points in common with the feasible region. (Why?)

Example 5 Minimise Z = 3x + 2y

subject to the constraints:

$$x + y \geq 8 \quad \text{... (1)}$$

$$3x + 5y \leq 15 \quad \text{... (2)}$$

$$x \geq 0,\ y \geq 0 \quad \text{... (3)}$$

Solution Let us graph the inequalities (1) to (3) (Fig 12.6). Is there any feasible region? Why is so?

From Fig 12.6, you can see that there is no point satisfying all the constraints simultaneously. Thus, the problem is having no feasible region and hence no feasible solution.

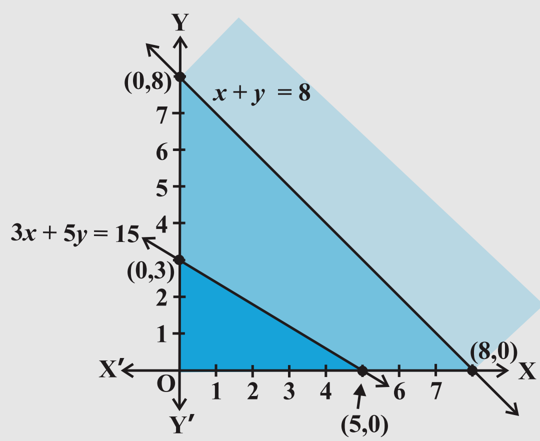


Fig 12.6

Remarks From the examples which we have discussed so far, we notice some general features of linear programming problems:

(i) The feasible region is always a convex region.

(ii) The maximum (or minimum) solution of the objective function occurs at the vertex (corner) of the feasible region. If two corner points produce the same maximum (or minimum) value of the objective function, then every point on the line segment joining these points will also give the same maximum (or minimum) value.