

Question -

The coefficients of three consecutive terms of  $(1+x)^{n+5}$  are in the ratio 5 : 10 : 14. Then,  $n$  is equal to (2013 Adv.)

Ans - 6

Solution -

Let the three consecutive terms in  $(1+x)^{n+5}$  be  $t_r, t_{r+1}, t_{r+2}$  having coefficients  ${}^{n+5}C_{r-1}, {}^{n+5}C_r, {}^{n+5}C_{r+1}$ .

Given,  ${}^{n+5}C_{r-1} : {}^{n+5}C_r : {}^{n+5}C_{r+1} = 5 : 10 : 14$

$$\therefore \frac{{}^{n+5}C_r}{{}^{n+5}C_{r-1}} = \frac{10}{5} \text{ and } \frac{{}^{n+5}C_{r+1}}{{}^{n+5}C_r} = \frac{14}{10}$$

$$\Rightarrow \frac{n+5-(r-1)}{r} = 2 \text{ and } \frac{n-r+5}{r+1} = \frac{7}{5}$$

$$\Rightarrow n-r+6=2r \text{ and } 5n-5r+25=7r+7$$

$$\Rightarrow n+6=3r \text{ and } 5n+18=12r$$

$$\therefore \frac{n+6}{3} = \frac{5n+18}{12}$$

$$\Rightarrow 4n+24=5n+18 \Rightarrow n=6$$