

Question -

Let  $n$  be a positive integer. If the coefficients of 2nd, 3rd, and 4th terms in the expansion of  $(1 + x)^n$  are in AP, then the value of  $n$  is... . (1994, 2M)

Ans - 7

Solution -

Let the coefficients of 2nd, 3rd and 4th terms in the expansion of  $(1 + x)^n$  is  ${}^n C_1, {}^n C_2, {}^n C_3$ .

According to given condition,

$$2({}^n C_2) = {}^n C_1 + {}^n C_3$$
$$\Rightarrow 2 \frac{n(n-1)}{1 \cdot 2} = n + \frac{n(n-1)(n-2)}{1 \cdot 2 \cdot 3}$$

$$\Rightarrow n - 1 = 1 + \frac{(n-1)(n-2)}{6}$$

$$\Rightarrow n - 1 = 1 + \frac{n^2 - 3n + 2}{6}$$

$$\Rightarrow 6n - 6 = 6 + n^2 - 3n + 2$$

$$\Rightarrow n^2 - 9n + 14 = 0$$

$$\Rightarrow (n-2)(n-7) = 0$$

$$\Rightarrow n = 2$$

or  $n = 7$

But  ${}^n C_3$  is true for  $n \geq 3$ , therefore  $n = 7$  is the answer.