Question -

The ratio of the 5th term from the beginning to the 5th term from the end in the binomial expansion of

$$\left(2^{\frac{1}{3}} + \frac{1}{2(3)^{\frac{1}{3}}}\right)^{10}$$
 is (2019 Main, 12 Jan I)

(a) 1:
$$2(6)^{\frac{1}{3}}$$
 (b) 1: $4(16)^{\frac{1}{3}}$ (c) $4(36)^{\frac{1}{3}}$: 1 (d) $2(36)^{\frac{1}{3}}$: 1

Ans - C Solution -

Since, rth term from the end in the expansion of a binomial $(x + a)^n$ is same as the (n - r + 2)th term from the beginning in the expansion of same binomial.

$$\therefore \text{Required ratio} = \frac{T_5}{T_{10-5+2}} = \frac{T_5}{T_7} = \frac{T_{4+1}}{T_{6+1}}$$

$$\Rightarrow \frac{T_5}{T_{10-5+2}} = \frac{{}^{10}C_4(2^{1/3})^{10-4} \left(\frac{1}{2(3)^{1/3}}\right)^4}{{}^{10}C_6(2^{1/3})^{10-6} \left(\frac{1}{2(3)^{1/3}}\right)^6}$$

$$[\because T_{r+1} = {}^{n}C_r x^{n-r} a^r]$$

$$= \frac{2^{6/3} (2(3)^{1/3})^6}{2^{4/3} (2(3)^{1/3})^4} \qquad [\because {}^{10}C_4 = {}^{10}C_6]$$

$$= 2^{6/3-4/3} (2(3)^{1/3})^{6-4}$$

$$= 2^{2/3} \cdot 2^2 \cdot 3^{2/3} = 4(6)^{2/3} = 4(36)^{1/3}$$

So, the required ratio is $4(36)^{1/3}:1$.