Question -

If some three consecutive coefficients in the binomial expansion of $(x+1)^n$ in powers of x are in the ratio 2 : 15 : 70, then the average of these three coefficients is

(2019 Main, 9 April II)

(a) 964 (b) 227 (c) 232 (d) 625

Ans - C

Solution -

Given binomial is $(x+1)^n$, whose general term, is $T_{r+1} = {}^nC_r\, x^r$

According to the question, we have

$$ {}^nC_{r-1} : {}^nC_r : {}^nC_{r+1} = 2 : 15 : 70 $$

Now, $$\frac{{}^nC_{r-1}}{{}^nC_r} = \frac{2}{15}$$

$$\Rightarrow \quad \frac{\dfrac{n!}{(r-1)!(n-r+1)!}}{\dfrac{n!}{r!(n-r)!}} = \frac{2}{15}$$

$$\Rightarrow \quad \frac{r}{n-r+1} = \frac{2}{15} \Rightarrow 15r = 2n - 2r + 2$$

$$\Rightarrow \quad 2n - 17r + 2 = 0 \qquad \ldots\text{(i)}$$

Similarly, $$\frac{{}^nC_r}{{}^nC_{r+1}} = \frac{15}{70} \Rightarrow \frac{\dfrac{n!}{r!(n-r)!}}{\dfrac{n!}{(r+1)!(n-r-1)!}} = \frac{3}{14}$$

$$\Rightarrow \quad \frac{r+1}{n-r} = \frac{3}{14} \Rightarrow 14r + 14 = 3n - 3r$$

$$\Rightarrow \quad 3n - 17r - 14 = 0 \qquad \ldots\text{(ii)}$$

On solving Eqs. (i) and (ii), we get

$n - 16 = 0 \Rightarrow n = 16$ and $r = 2$

Now, the average $= \dfrac{{}^{16}C_1 + {}^{16}C_2 + {}^{16}C_3}{3}$

$$= \frac{16 + 120 + 560}{3} = \frac{696}{3} = 232$$