

Question -

The value of r for which

$${}^{20}C_r \cdot {}^{20}C_0 + {}^{20}C_{r-1} \cdot {}^{20}C_1 + {}^{20}C_{r-2} \cdot {}^{20}C_2 + \dots + {}^{20}C_0 \cdot {}^{20}C_r$$

is maximum, is

(2019 Main, 11 Jan I)

- (a) 15 (b) 10 (c) 11 (d) 20

Ans - D

Solution -

We know that,

$$\begin{aligned} (1+x)^{20} &= {}^{20}C_0 + {}^{20}C_1x + {}^{20}C_2x^2 + \dots + \\ &{}^{20}C_{r-1}x^{r-1} + {}^{20}C_r x^r + \dots + {}^{20}C_{20}x^{20} \\ \therefore (1+x)^{20} \cdot (1+x)^{20} &= ({}^{20}C_0 + {}^{20}C_1x + \\ &{}^{20}C_2x^2 + \dots + {}^{20}C_{r-1}x^{r-1} + {}^{20}C_r x^r + \dots + {}^{20}C_{20}x^{20}) \\ &\times ({}^{20}C_0 + {}^{20}C_1x + \dots + {}^{20}C_{r-1}x^{r-1} + {}^{20}C_r x^r \\ &\quad + \dots + {}^{20}C_{20}x^{20}) \\ \Rightarrow (1+x)^{40} &= ({}^{20}C_0 \cdot {}^{20}C_r + {}^{20}C_1 \cdot {}^{20}C_{r-1} \dots \\ &\quad \dots + {}^{20}C_r \cdot {}^{20}C_0) x^r + \dots \end{aligned}$$

On comparing the coefficient of x^r of both sides, we get

$${}^{20}C_0 \cdot {}^{20}C_r + {}^{20}C_1 \cdot {}^{20}C_{r-1} + \dots + {}^{20}C_r \cdot {}^{20}C_0 = {}^{40}C_r$$

The maximum value of ${}^{40}C_r$ is possible only when $r = 20$

[$\because {}^nC_{n/2}$ is maximum when n is even]

Thus, required value of r is 20.