Question -

The sum of coefficients of integral powers of x in the binomial expansion $(1-2\sqrt{x})^{50}$ is **(2015 Main)**

(a) $\frac{1}{2}(3^{50}+1)$ (b) $\frac{1}{2}(3^{50})$ (c) $\frac{1}{2}(3^{50}-1)$ (d) $\frac{1}{2}(2^{50}+1)$

Ans - A

Solution -

We have,

$$(1-2\sqrt{x})^{50} = C_o - C_1 2\sqrt{x} + C_2(\sqrt{2}x)^2 + \ldots + C_{50}(2\sqrt{x})^{50} \quad \ldots\text{(i)}$$

$$(1+2\sqrt{x})^{50} = C_o + C_1 2\sqrt{x} + C_2(2\sqrt{x})^2 + \ldots + C_{50}(2\sqrt{x})^{50} \quad \ldots\text{(ii)}$$

On adding Eqs. (i) and (ii), we get

$$(1-2\sqrt{x})^{50} + (1+2\sqrt{x})^{50} = 2\,[C_0 + C_2(2\sqrt{x})^2 + \ldots + C_{50}(2\sqrt{x})^{50}] \quad \ldots\text{(iii)}$$

$$\Rightarrow \quad \frac{(1-2\sqrt{x})^{50} + (1+2\sqrt{x})^{50}}{2} = C_0 + C_2(2\sqrt{x})^2 + \ldots + C_{50}(2\sqrt{x})^{50}$$

On putting $x=1$, we get

$$\frac{(1-2\sqrt{1})^{50} + (1+2\sqrt{1})^{50}}{2} = C_0 + C_2 + \ldots + C_{50}(2)^{50}$$

$$\Rightarrow \quad \frac{(-1)^{50} + (3)^{50}}{2} = C_0 + C_2(2)^2 + \ldots + C_{50}(2)^{50}$$

$$\Rightarrow \quad \frac{1+3^{50}}{2} = C_0 + C_2(2)^2 + \ldots + C_{50}(2)^{50}$$