Properties related to ^nC_r :

(i) $^nC_r = {}^nC_{n-r}$

Note : If $^nC_x = {}^nC_y \Rightarrow$ Either $x = y$ or $x + y = n$

(ii) $^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$

(iii) $\dfrac{^nC_r}{^nC_{r-1}} = \dfrac{n-r+1}{r}$

(iv) $^nC_r = \dfrac{n}{r}\,{}^{n-1}C_{r-1} = \dfrac{n(n-1)}{r(r-1)}\,{}^{n-2}C_{r-2} = \ldots\ldots\ldots\ldots = \dfrac{n(n-1)(n-2)\ldots\ldots\ldots(n-(r-1))}{r\,(r-1)(r-2)\ldots\ldots.2\,.1}$

(v) If n and r are relatively prime, then ^nC_r is divisible by n. But converse is not necessarily true.

Some standard expansions :

(i) Consider the expansion

$$(x + y)^n = \sum_{r=0}^{n} {}^nC_r\ x^{n-r}\, y^r = {}^nC_0\, x^n\, y^0 + {}^nC_1\, x^{n-1}\, y^1 + \ldots\ldots\ldots.+ {}^nC_r\, x^{n-r}\, y^r + \ldots\ldots\ldots.+ {}^nC_n\, x^0\, y^n \quad \ldots.(i)$$

(ii) Now replace $y \to -y$ we get

$$(x - y)^n = \sum_{r=0}^{n} {}^nC_r\, (-1)^r\, x^{n-r}\, y^r = {}^nC_0\, x^n\, y^0 - {}^nC_1\, x^{n-1}\, y^1 + \ldots + {}^nC_r\, (-1)^r x^{n-r}\, y^r + \ldots + {}^nC_n\, (-1)^n\, x^0\, y^n \quad \ldots.(ii)$$

(iii) Adding (i) & (ii), we get
$(x + y)^n + (x - y)^n = 2[{}^nC_0\, x^n\, y^0 + {}^nC_2\, x^{n-2}\, y^2 + \ldots\ldots\ldots]$

(iv) Subtracting (ii) from (i), we get
$(x + y)^n - (x - y)^n = 2[{}^nC_1\, x^{n-1}\, y^1 + {}^nC_3\, x^{n-3}\, y^3 + \ldots\ldots\ldots]$