Question -

The value of $({}^{21}C_1 - {}^{10}C_1) + ({}^{21}C_2 - {}^{10}C_2) + ({}^{21}C_3 - {}^{10}C_3) + ({}^{21}C_4 - {}^{10}C_4) + \ldots + ({}^{21}C_{10} - {}^{10}C_{10})$ is **(2017 Main)**

(a) $2^{21} - 2^{11}$ (b) $2^{21} - 2^{10}$

(c) $2^{20} - 2^9$ (d) $2^{20} - 2^{10}$

Ans - D

Solution -

$$({}^{21}C_1 - {}^{10}C_1) + ({}^{21}C_2 - {}^{10}C_2) + ({}^{21}C_3 - {}^{10}C_3) + \ldots + ({}^{21}C_{10} - {}^{10}C_{10})$$

$$= ({}^{21}C_1 + {}^{21}C_2 + \ldots + {}^{21}C_{10}) - ({}^{10}C_1 + {}^{10}C_2 + \ldots + {}^{10}C_{10})$$

$$= \frac{1}{2}({}^{21}C_1 + {}^{21}C_2 + \ldots + {}^{21}C_{20}) - (2^{10} - 1)$$

$$= \frac{1}{2}({}^{21}C_1 + {}^{21}C_2 + \ldots + {}^{21}C_{21} - 1) - (2^{10} - 1)$$

$$= \frac{1}{2}(2^{21} - 2) - (2^{10} - 1) = 2^{20} - 1 - 2^{10} + 1 = 2^{20} - 2^{10}$$