Question -

The positive value of λ for which the coefficient of x^2 in the expression $x^2\left(\sqrt{x}+\frac{\lambda}{x^2}\right)^{10}$ is 720, is

(2019 Main, 10 Jan II)

(a) 3 (b) $\sqrt{5}$ (c) $2\sqrt{2}$ (d) 4

Ans - D

Solution -

The general term in the expansion of binomial expression $(a+b)^n$ is $T_{r+1} = {}^nC_r\, a^{n-r} b^r$, so the general term in the expansion of binomial expression $x^2\left(\sqrt{x}+\frac{\lambda}{x^2}\right)^{10}$ is

$$T_{r+1} = x^2\left({}^{10}C_r(\sqrt{x})^{10-r}\left(\frac{\lambda}{x^2}\right)^r\right) = {}^{10}C_r\ x^2 \cdot x^{\frac{10-r}{2}}\ \lambda^r\ x^{-2r}$$

$$= {}^{10}C_r\, \lambda^r\ x^{2+\frac{10-r}{2}-2r}$$

Now, for the coefficient of x^2, put $2+\frac{10-r}{2}-2r=2$

$$\Rightarrow \qquad \frac{10-r}{2}-2r=0$$

$$\Rightarrow \qquad 10-r=4r \Rightarrow r=2$$

So, the coefficient of x^2 is ${}^{10}C_2\,\lambda^2 = 720$ [given]

$$\Rightarrow \qquad \frac{10!}{2!8!}\lambda^2 = 720 \quad \Rightarrow \frac{10\cdot 9\cdot 8!}{2\cdot 8!}\lambda^2 = 720$$

$$\Rightarrow \qquad 45\,\lambda^2 = 720$$

$$\Rightarrow \qquad \lambda^2 = 16 \quad \Rightarrow \lambda = \pm 4$$

$$\therefore \qquad \lambda = 4 \qquad [\lambda > 0]$$