Question -

The sum of the real values of x for which the middle term in the binomial expansion of $\left(\dfrac{x^3}{3} + \dfrac{3}{x}\right)^8$ equals 5670 is

(2019 Main, 11 Jan I)

(a) 4   (b) 0   (c) 6   (d) 8

Ans - B

Solution -

In the expansion of $\left(\dfrac{x^3}{3} + \dfrac{3}{x}\right)^8$, the middle term is T_{4+1}.

$[\because$ Here, $n = 8$, which is even, therefore middle term $= \left(\dfrac{n+2}{2}\right)$th term$]$

$\therefore \quad 5670 = {}^8C_4 \left(\dfrac{x^3}{3}\right)^4 \left(\dfrac{3}{x}\right)^4 = \dfrac{8 \cdot 7 \cdot 6 \cdot 5}{1 \cdot 2 \cdot 3 \cdot 4} x^8$

$\left[\because T_{r+1} = {}^8C_r \left(\dfrac{x^3}{3}\right)^{8-r} \left(\dfrac{3}{x}\right)^r\right]$

$\Rightarrow \quad x^8 = 3^4 \Rightarrow x = \pm\sqrt{3}$

So, sum of all values of x i.e $+\sqrt{3}$ and $-\sqrt{3} = 0$