

According to the theorem, it is possible to expand the power $(x + y)^n$ into a sum involving terms of the form $ax^b y^c$, where the exponents b and c are nonnegative integers with $b + c = n$, and the coefficient a of each term is a specific positive integer depending on n and b . When an exponent is zero, the corresponding power is usually omitted from the term (so that $3x^2 y^0$ would be written as $3x^2$).

For example, consider the following expansion:

$$(x + y)^4 = x^4 + 4x^3 y + 6x^2 y^2 + 4xy^3 + y^4$$

Any coefficient a in a term $ax^b y^c$ of the expanded version is known as a binomial coefficient. The binomial coefficient also arises in combinatorics, where it gives the number of different combinations of b elements that can be chosen from a set of n elements. Recall that this could be written with the notation $\binom{n}{b}$, or “ n choose b .”

According to the binomial theorem, it is possible to expand any power of $x + y$ into a sum of the form:

$$(x + y)^n = \binom{n}{0} x^n y^0 + \binom{n}{1} x^{n-1} y^1 + \binom{n}{2} x^{n-2} y^2 + \dots + \binom{n}{n-1} x^1 y^{n-1} + \binom{n}{n} x^0 y^n$$

where each value $\binom{n}{k}$ is a specific positive integer known as binomial coefficient. This formula is referred to as the Binomial Formula. Using summation notation, it can be written as:

$$(x + y)^n = \sum_{k=0}^n \binom{n}{k} x^{n-k} y^k = \sum_{k=0}^n \binom{n}{k} x^k y^{n-k}$$

Pascal's triangle is an alternative way of determining the coefficients that arise in binomial expansions, using a diagram rather than algebraic methods. For a binomial expansion with a relatively small exponent, this can be a straightforward way to determine the coefficients.

The rows of Pascal's triangle are numbered, starting with row $n = 0$ at the top. The entries in each row are numbered from the left beginning with $k = 0$ and are usually staggered relative to the numbers in the adjacent rows. A simple construction of the triangle proceeds in the following manner. On row 0, write only the number 1. Then, to construct the elements of following rows, add the two above numbers to find the new value. If either of the above numbers is not present, substitute a zero in its place. For example, each number in row one is $0 + 1 = 1$.

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1