4. Suppose that the foci of the ellipse $\frac{x^2}{9}+\frac{y^2}{5}=1$ are $(f_1, 0)$ and $(f_2, 0)$ where $f_1 > 0$ and $f_2 < 0$. Let P_1 and P_2 be two parabolas with a common vertex at (0, 0) and with foci at $(f_1, 0)$ and $(2f_2, 0)$, respectively. Let T_1 be a tangent to P_1 which passes through $(2f_2, 0)$ and T_2 be a tangent to P_2 which passes through $(f_1, 0)$. If m_1 is the slope of T_1 and m_2 is the slope of T_2, then the value of $\left(\frac{1}{m_1^2}+m_2^2\right)$ is

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Solution: -

4. **(4)** Ellipse: $\frac{x^2}{9}+\frac{y^2}{5}=1$

$\Rightarrow \quad a=3, b=\sqrt{5}$ and $e=\frac{2}{3}$

$\therefore \quad f_1=2$ and $f_2=-2$

$P_1: y^2=8x$ and $P_2: y^2=-16x$

$T_1: y=m_1x+\frac{2}{m_1}$

It passes through (–4, 0),

$0=-4m_1+\frac{2}{m_1} \Rightarrow m_1^2=\frac{1}{2}$

$T_2: y=m_2x-\frac{4}{m_2}$

It passes through (2, 0)

$0=2m_2-\frac{4}{m_2} \Rightarrow m_2^2=2$

$\therefore \quad \frac{1}{m_1^2}+m_2^2=4$